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ON THE COLLISIONAL HYPOTHESIS OF THE ORIGIN OF THE PERSEID METEOR STREAM

Abstract: Evidences for a collisional origin of the Perseid meteor stream, as maintained by Guigay, are discussed. It is shown that the approaches of several cometary orbits to those of the Perseids do not support the opinion of their common origin, being fully explainable by a pure chance. Moreover, the investigation of the dynamical conditions of the hypothetical collision, based on the determination of the relative velocities of the concerned bodies in the region of their nearest approach, leads to entirely unacceptable results.

A general theory of the random distribution of known cometary orbits around an arbitrary point in the space is outlined and the due numerical results are derived. Using the present data other similar problems may be treated to find out whether a grouping of cometary orbits towards any point may be explained by means of the Law of Chance or not.

1. Introduction

In his paper on the constitution of the Perseid meteor stream Guigay expressed the opinion that the stream was generated by a sudden rupture of the parent comet caused by a collision with another body; at that collision five known comets are said to have come into existence. The main argument supporting this conclusion consists in an approach of the orbits of the five comets in question around a point in the space in which the orbits of Perseids observed in different nights approximatively intersect. The ecliptical co-ordinates of that point are

$l = 28°57'$

$b = +65°15'$

$r = 1.217,$

and the comets concerned: 1825 II, 1826 V, 1862 III (the parent comet of the stream), 1877 III, 1909 I, and 1932 V.

More recently, several authors [2, 3] have shown that the enormous dispersion of the stream may well be attributed to planetary perturbations dispersing continually the originally huge meteor swarm. For this explanation, high original relative velocities are not required; just the opposite—they may be scarcely admitted. Now the question remains how to explain the peculiar grouping of the orbits, found by Guigay, and whether it may be taken for a matter of chance only.

2. The Dynamical Conditions of the Hypothetical Collision

For a deeper insight into the dynamical conditions of the hypothetical collision, the relative velocities of the comets near the point of separation are of a particular interest. These velocities may be obtained directly from the orbital elements. For the sake of simplification of the numerical computations, we shall assume parabolical orbits in all cases. As the point of the nearest approach of the orbits lies in a not too great heliocentric distance (r ranging from 0.95 to 1.38 in individual cases), and the major axes of the orbits are considerable ($a = 24.3$ and more), the errors introduced with this simplification are negligible for our purpose. In the resulting velocity they amount to about 1% for comets 1862 III, 1909 I, and 1932 V, and are even substantially lower in the remaining three cases. Referring the velocities to the rectangular system of ecliptical co-ordinates, we obtain for the velocity components the following relations:

$$V_x = \frac{dx}{dt} = \frac{V_0}{\sqrt{2q}} [- (\sin \omega + \sin u) \cos \Omega - (\cos \omega + \cos u) \sin \Omega \cos i]$$

$$V_y = \frac{dy}{dt} = \frac{V_0}{\sqrt{2q}} [- (\sin \omega + \sin u) \sin \Omega + (\cos \omega + \cos u) \cos \Omega \cos i] \qquad (1)$$

$$V_z = \frac{dz}{dt} = \frac{V_0}{\sqrt{2q}} (\cos \omega + \cos u) \sin i$$

Here V_0 denotes the circular velocity for $r = 1$ (29.8 km/s), u the argument of the latitude ($u = v + \omega$) and other symbols the usual orbital elements. In Table X of the quoted paper by Guigay, the true anomalies v of the five comets and the true anomaly v' of Comet 1862 III may be found, in which the orbits approach to a minimum distance one from another. Substituting these values together with the due orbital elements into (1), we find the rectangular velocity components of the five comets (V_x, V_y, V_z) as well those of Comet 1862 III (V'_x, V'_y, V'_z). Then the relative velocities W of each of the five comets with respect to Comet 1862 III in the point of the closest possible approach are

$$W = \sqrt{(V_x - V'_x)^2 + (V_y - V'_y)^2 + (V_z - V'_z)^2} \qquad (2)$$

The results of the computation are summarized in Table I. It is shown that the relative velocities are high, ranging from 35 km/s to 85 km/s with an average at 60 km/s, whereas the parabolical velocity at $r = 1.217$ is 38 km/s. The velocities of separation which have hitherto been observed on different occasions (the disintegration of Comet Biela, 1882 II, 1947 XII, 1955 g) are three to four orders lower. Even if we were to assume the possibility of a rupture with such enormous relative velocities of the separated comets, which is entirely unbelievable, the resulting semi-major axes would be dispersed in an absolutely different way than the observations indicate. A great proportion of orbits would be turned into hyperbolas, and only for a negligible fraction the orbits of $1/a$ within the limits 0.00 to 0.04, as actually observed, could arise. A rough computation shows that if a body is ejected at the given point from Comet 1862 III with an initial velocity of 60 km/s and if the ejections in each direction are equally probable, only four orbits out of 1000 would result, for which $0.00 < 1/a < 0.04$! Taking into account the fact that the group of six comets observed during the last two centuries must represent—with regard to the long periods of revolution—only a small fraction of the true total number, the required character of the original collision and rupture comes out absolutely inadmissible.

Table I

Comet	1825 II	1826 V	1877 III	1909 I	1932 V
v	−59.°0	−164.°0	−47.°8	+39.°0	+32.°0
v'	−47.°5	− 67.°0	−56.°8	−2.°8	−51.°5
V_x	+20.2	−5.8	−37.2	+9.5	−29.0
V_y	+5.0	−9.2	+8.9	+28.5	+16.1
V_z	−33.0	−34.1	−1.9	+31.0	+21.9
V'_x	+15.8	+18.3	+17.2	+5.0	+16.4
V'_y	−27.9	−26.3	−27.3	−25.0	−27.7
V'_z	−22.7	−16.0	−19.5	−34.5	−21.3
$V_x - V'_x$	+4.4	−24.1	−54.4	+4.6	−45.4
$V_y - V'_y$	+32.9	+17.1	+36.3	+53.5	+43.8
$V_z - V'_z$	−10.3	−18.1	−17.6	+65.5	+43.2
W	34.8	34.6	67.7	84.7	76.5

Table II

Comet	1825 II	1826 V	1877 III	1909 I	1932 V
i	89.°7	90.°6	77.°2	52.°1	71.°7
$r_{☊}$	(1600)	0.046	3.66	0.84	1.16
$r_{☋}$	0.88	0.065	1.39	(190)	9.56

It may be emphasized that the wide variety of the six orbits concerned can by no means be explained by the planetary perturbations having changed the orbits after the separation had taken place at considerably lower relative velocities. From Table II it is seen that the orbital inclinations of all these comets are high, and that their nodes lie in considerable distances from the orbits of the planets, except Comet 1932 V which passes the descending node not far from the orbit of Saturn. Moreover, such strong planetary perturbations as were required would have dispersed the orbits in such way that they could not more approach to the original point of separation and would bring about an additional dispersion of $1/a$.

From all what has been said above it is obvious that the hypothesis of a common origin of the six comets 1825 II, 1826 V, 1862 III, 1877 III, 1909 I, and 1932 V must be rejected anyway.

3. *The General Problem of Random Approaches of the Known Cometary Orbits*

As the problem of the grouping of orbits is of general interest, it will be treated here in a more general form. The basic question is, what is the a priori probability that a given number of orbits of known comets will approach to an arbitrary point of space within a given distance. The solution of the problem may be substantially simplified by introducing the following assumptions:

(1) The distribution of the directions of cometary perihelia with respect to the Sun is isotropical. As a matter of fact, this assumption is approximatively valid when omitting the short-periodic comets, particularly those of Jupiter's family, exhibiting a striking ecliptical concentration. The restriction of our examinations to the long-periodic and nearly parabolical orbits seems to be quite reasonable, and cannot anyway influence the results as to the particular problem of Perseids: the point of the hypothetical collision lies, as a matter of fact, in a high ecliptical latitude ($b = 65°$), entirely avoided by the short-periodic orbits.

(2) All concerned orbits are parabolical. As far as the short-periodic comets are omitted, this simplification is usable. The hyperbolical orbits and the elliptical orbits of high eccentricities depart in moderate heliocentric distances but little from parabolas of the same perihelion distances. Again, for our particular case of $r = 1.217$ the assumption is sound.

(3) The minimum distance in which the approaching orbits are searched is small compared with the heliocentric distance of the point of approach. For different reasons it is advisable to restrict the examination to a distance of about 0.1 astr. units only; approaches within a still greater distance are of no special interest.

From geometrical considerations we find for the a priori probability P_R that a point, selected at random within a heliocentric distance R will lie within a distance Δ from a parabolical orbit of perihelion distance q, the following expression:

$$P_R = \frac{3\Delta^2}{2R^3} \int_q^R \sqrt{1 + r^2 \left(\frac{\mathrm{d}v}{\mathrm{d}r}\right)^2}\, \mathrm{d}r \tag{3}$$

where

$$\frac{\mathrm{d}v}{\mathrm{d}r} = \frac{1}{r} \sqrt{\frac{q}{r-q}} \tag{4}$$

Inserting (4) into (3) we obtain:

$$P_R = \frac{3\Delta^2}{2R^3} \int_q^R \sqrt{\frac{r}{r-q}}\, \mathrm{d}r \tag{5}$$

By means of trigonometric substitutions and further adaptations (5) can be integrated as follows:

$$P_R = \frac{3\Delta^2}{2R^2}\left[\sqrt{1-\frac{q}{R}} + \frac{q}{R}\log_e\left(\sqrt{\frac{R}{q}} + \sqrt{\frac{R}{q}-1}\right)\right] \tag{6}$$

Formula (6) may be expressed with the aid of the Gudermannians in a form more convenient for numerical computations, as far as the values of the inverse Gudermannians may be found in proper tables, e. g. (4). We have:

$$P_R = \frac{3\Delta^2}{2R^2}\left[\sqrt{1-\frac{q}{R}} + \frac{q}{R}\,\mathrm{gd}^{-1}\cos^{-1}\sqrt{\frac{q}{R}}\right] \tag{7}$$

Now we shall examine the term in square brackets. For the sake of simplification we shall put

$$\lambda = \frac{q}{R} \tag{8}$$

$$\mu = \sqrt{1-\frac{q}{R}} \tag{9}$$

$$\nu = \frac{q}{R}\log_e\left(\sqrt{\frac{R}{q}} + \sqrt{\frac{R}{q}-1}\right) \tag{10}$$

For a uniform distribution of perihelion distances, i. e.

$$n(q)\,\mathrm{d}q = \text{const.} \tag{11}$$

we obtain the mean values of μ and ν, respectively, as follows:

$$\bar{\mu} = \frac{1}{R}\int_0^R \sqrt{1-\frac{q}{R}}\,\mathrm{d}q = \int_0^1 \sqrt{1-\lambda}\,\mathrm{d}\lambda = \frac{2}{3} \tag{12}$$

$$\bar{\nu} = \frac{1}{R^2}\int_0^R q\log_e\left(\sqrt{\frac{R}{q}} + \sqrt{\frac{R}{q}-1}\right)\mathrm{d}q = \int_0^1 \lambda\log_e\frac{1+\sqrt{1-\lambda}}{\sqrt{\lambda}}\,\mathrm{d}\lambda = \frac{1}{3} \tag{13}$$

According to (6), (12), and (13) the mean value of the probability P_R under the assumption (11) is

$$\bar{P}_R = \frac{3\Delta^2}{2R^2} \tag{14}$$

If there are n_R comets with the perihelion distances $q < R$, the expected number of comets passing the point within a distance Δ comes out

$$N_R = \frac{3\Delta^2 n_R}{2R^2} \tag{15}$$

However, the distribution of perihelion distances is actually far from being uniform, and it is necessary to decide how much the results become altered by accepting the actual distribution law, morecomplicated and essentially different from (11). It may be noted that the sums $\mu + \nu$, which are functions of the ratio q/R alone, represent the relative probabilities P_R for the orbits of a given q/R.

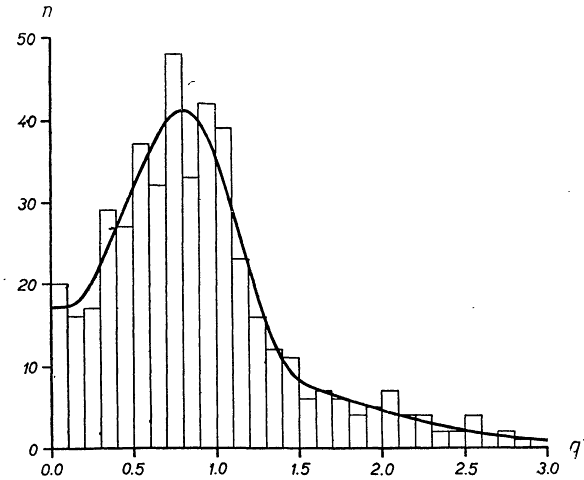

Figure 1. The observed and smoothed distribution of known long-periodic comets according to the perihelion distances.

From formula (6) the mean values of μ, ν for different intervals of q/R may be found by integrating between the due limits q_1, q_2, for which

$$0 \leqq \frac{q_1}{R} = \lambda_1 < \frac{q_2}{R} = \lambda_2 \leqq 1 \tag{16}$$

Thus we obtain:

$$\bar{\mu}_{12} = \frac{1}{\lambda_2 - \lambda_1} \int\limits_{\lambda_1}^{\lambda_2} \sqrt{1-\lambda}\, d\lambda = \frac{2}{3} \cdot \frac{(1-\lambda_1)^{\frac{3}{2}} - (1-\lambda_2)^{\frac{3}{2}}}{\lambda_2 - \lambda_1} \tag{17}$$

and

$$\bar{\nu}_{12} = \frac{1}{\lambda_2 - \lambda_1} \int\limits_{\lambda_1}^{\lambda_2} \lambda \log_e \frac{1 + \sqrt{1-\lambda}}{\sqrt{\lambda}}\, d\lambda = \frac{1}{2(\lambda_2 - \lambda_1)} \left(\lambda_2^2 \log_e \frac{1 + \sqrt{1-\lambda_2}}{\sqrt{\lambda_2}} - \lambda_1^2 \log_e \frac{1+\sqrt{1-\lambda_1}}{\sqrt{\lambda_1}} \right) - \frac{(2+\lambda_2)\sqrt{1-\lambda_2} - (2+\lambda_1)\sqrt{1-\lambda_1}}{6(\lambda_2 - \lambda_1)} \tag{18}$$

The course of the function $P_R/\bar{P}_R = \mu + \nu$ is shown in Table III and Figure 2. We see that $P_R/\bar{P}_R = 1$ if $\lambda = 0$ or $\lambda = 0.681$; it attains the maximum value of 1.200 at $\lambda = 0.305$ and drops to zero at $\lambda = 1$. It is essential, however, that within more than 4/5 of the interval from $\lambda = 0$ to $\lambda = 1$, $P_R/\bar{P}_R$ differs from one by less than 20%.

Table III

λ	μ	ν	$\frac{P_R}{\bar{P}_R}$
0.00	1.000	0.000	1.000
0.05	0.975	0.109	1.084
0.10	0.949	0.182	1.130
0.15	0.922	0.240	1.162
0.20	0.894	0.289	1.183
0.25	0.866	0.329	1.195
0.30	0.837	0.363	1.200
0.35	0.806	0.391	1.197
0.40	0.775	0.413	1.187
0.45	0.742	0.429	1.171
0.50	0.707	0.441	1.148
0.55	0.671	0.447	1.118
0,60	0.632	0.447	1.080
0.65	0.592	0.442	1.034
0.70	0.548	0.431	0.978
0.75	0.500	0.412	0.912
0.80	0.447	0.385	0.832
0.85	0.387	0.347	0.735
0.90	0.316	0.295	0.611
0.95	0.224	0.216	0.440
1.00	0.000	0.000	0.000

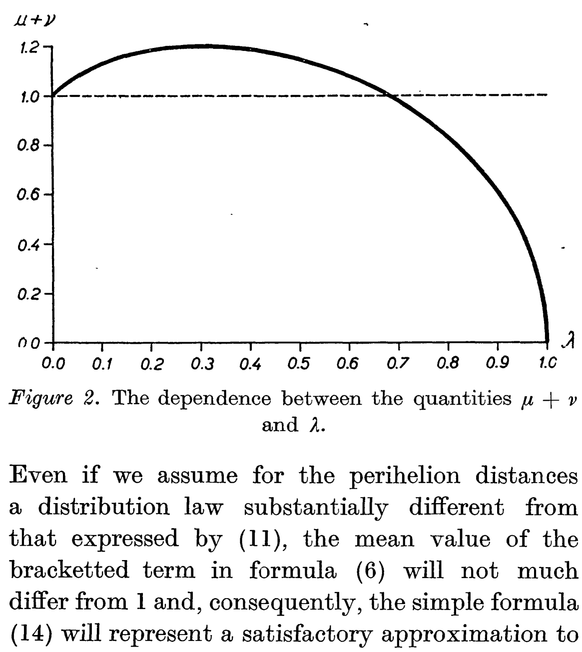

Figure 2. The dependence between the quantities $\mu + \nu$ and λ.

Even if we assume for the perihelion distances a distribution law substantially different from that expressed by (11), the mean value of the bracketted term in formula (6) will not much differ from 1 and, consequently, the simple formula (14) will represent a satisfactory approximation to the actual state. Selecting intentionally two peculiar distribution laws, in which the perihelion distances are uniformly distributed over the interval (0, $R/2$) or ($R/2$, R) only and avoid the other half of the interval (0, R), we obtain:

$$\frac{P_R}{\bar{P}_R} = 1.160 \qquad \text{for } 0 < q < \frac{R}{2}$$
$$\frac{P_R}{\bar{P}_R} = 0.840 \qquad \text{for } \frac{R}{2} < q < R \tag{19}$$

respectively.

For the actual distribution of perihelion distances, shown in Table IV, we obtain by numerical integration values which are still nearer to 1, e. g.

$$\frac{P_R}{\overline{P}_R} = 0.953 \qquad \text{for } R = 0.5$$

$$\frac{P_R}{\overline{P}_R} = 0.951 \qquad \text{for } R = 1.0$$

$$\frac{P_R}{\overline{P}_R} = 1.050 \qquad \text{for } R = 1.5 \qquad (20)$$

$$\frac{P_R}{\overline{P}_R} = 1.111 \qquad \text{for } R = 2.0$$

The fact that the expected number of approaches computed according to (14) differs from the correct value by a few per cent is unessential owing to small quantities concerned. It may be inferred that the number of approaches depends sensibly upon the number of comets, passing within the heliocentric distance of the point in question, but only minutely upon the distribution of their perihelia within this distance.

Formula (15) gives the mean expected number of approaches N_R inside a sphere of the radius R centred in the Sun. The corresponding number N'_R on the surface of the sphere may be obtained by differentiating as follows:

$$N'_R = \frac{N_{R+\mathrm{d}R}(R^3 + 3R^2\mathrm{d}R) - N_R R^3}{3R^2\mathrm{d}R} = \frac{\Delta^2}{2R^2}\left(n_R + R\frac{\mathrm{d}n_R}{\mathrm{d}R}\right) \qquad (21)$$

Not taking the differences between (5) and (15) into account, the probability of an approach would be independent of the heliocentric distance if

$$\frac{\mathrm{d}n}{\mathrm{d}r} = \frac{2n_R}{R} \qquad (22)$$

i. e. for a distribution law of the perihelion distances

$$n(q)\,\mathrm{d}q = \text{const} . q \qquad (23)$$

As the frequency of perihelion distances of known comets increases slower than required up to about $q = 0.8$ and even decreases for $q > 0.8$, the probability of an approach to a given distance becomes continually lower in the direction away from the Sun. In the region where no more perihelia occur, $\frac{\mathrm{d}n_R}{\mathrm{d}R}$ becomes zero and

$$N'_R = \frac{\Delta^2 n_R}{2R^2} = \frac{1}{3} N_R \qquad (24)$$

Owing to the simplifing conditions introduced above, the expression (24) indicates the lower limit of N'_R whereas the expression (15) is obviously

Table IV

R	n_0	n_R	$\frac{\mathrm{d}n_R}{\mathrm{d}R}$	$N_R\Delta^{-2}$	$N'_R\Delta^{-2}$
0.1	20	18	172	.	.
0.2	36	35	187	.	.
0.3	53	55	223	917	677
0.4	82	80	269	750	586
0.5	109	109	317	654	535
0.6	146	143	361	596	500
0.7	178	181	395	554	467
0.8	226	222	410	520	430
0.9	259	263	400	487	385
1.0	301	301	360	451	330
1.1	340	334	296	414	272
1.2	363	360	222	375	217
1.3	379	379	156	336	172
1.4	391	392	110	300	139
1.5	402	401	85	267	117
1.6	408	409	73	240	103
1.7	415	416	65	216	91
1.8	421	422	59	195	82
1.9	425	427	53	177	73
2.0	430	432	47	162	66
2.1	437	437	41	149	59
2.2	441	441	36	137	54
2.3	445	444	31	126	49
2.4	447	447	26	116	44
2.5	449	449	22	108	40
2.6	453	451	18	100	37
2.7	453	453	15	93	34
2.8	455	454	12	87	31
2.9	456	455	9	81	29
3.0	456	456	7	76	26

the upper limit. Hence, for an arbitrary heliocentric distance

$$\frac{1}{3} N_R < N'_R < N_R \qquad (25)$$

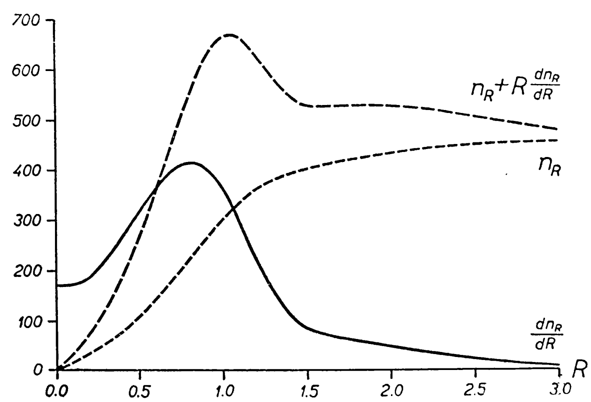

Figure 3. The course of the function n_R, its derivative, and $n_R + \frac{dn_R}{d_R}$ for the observed distribution of perihelion distances.

The principal formulae deduced in the text are evaluated in Table IV, and the courses of the functions concerned are graphically represented in Figures 2—4. As a source of reference, the General Catalogue of Cometary Orbits by Baldet and Obaldia [5] was used. All short-periodic comets with semi-major axes smaller than 10, i. e. those which due to the perturbing action of the planets show a considerable ecliptical concentration of orbits, and even in the region of the inner planets

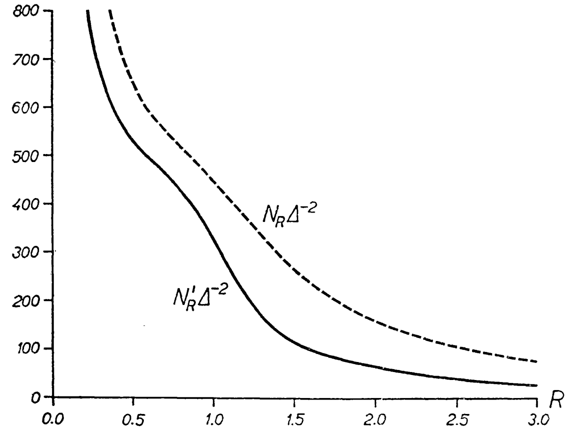

Figure 4. The expected number of known long-periodic cometary orbits N'_R (N_R) passing a point of heliocentric distance R (less than R) in a distance less than Δ, provided $\Delta \ll R$.

deviate appreciably from the parabolical motion, have been omitted. The first column of the Table contains the heliocentric distance R, the second column the actual number of known comets n_O with perihelion distances smaller than R, the third the smoothed number of these comets n_R, the fourth the change of the smoothed number with the heliocentric distance $\frac{\mathrm{d}n_R}{\mathrm{d}R}$, the fifth the quantity $N_R\Delta^{-2}$, and the sixth the quantity $N'_R\Delta^{-2}$. From the last two columns the expected number of approaches within a distance Δ may be obtained by multiplying the tabulated values by Δ^2.

A sample distribution of random approaches of

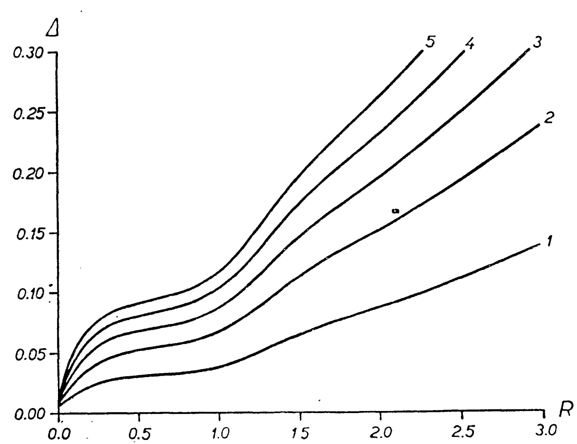

Figure 5. The expected distribution of the distances Δ from a point to the five nearest orbits of known long-periodic comets plotted against the heliocentric distance R.

the nearest passing orbits to a point situated in a heliocentric distance R may be derived by putting

$$\Delta_i = R\sqrt{\frac{2i-1}{n_R + R\frac{\mathrm{d}n_R}{\mathrm{d}R}}} \tag{26}$$

where Δ_1 denotes the distance of the nearest orbit, Δ_2 that of the second nearest one, etc. For selected values of R the sample distributions of the five nearest orbits are evaluated in Table V; the dependence of $\Delta_i (i = 1 \text{ to } 5)$ upon R is graphically represented in Figure 5.

Table V

R	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5
0.5	0.031	0.053	0.068	0.081	0.092
1.0	0.039	0.067	0.087	0.10	0.12
1.5	0.065	0.11	0.15	0.17	0.20
2.0	0.087	0.15	0.19	0.23	0.26
2.5	0.11	0.19	0.25	0.29	0.33
3.0	0.14	0.24	0.31	0.36	0.41

4. *The Grouping of Cometary Orbits in Guigay's Problem*

Now let us apply the outlined treatment to the particular problem of the origin of the Perseid meteor stream. Solving the above formulae for the heliocentric distance $R = 1.217$, in which the stream was formed according to Guigay, we obtain

$$n_R = 363$$

$$\frac{\mathrm{d}n_R}{\mathrm{d}R} = 210 \tag{27}$$

$$N'_R\Delta^{-2} = 209$$

In his paper Guigay makes use of the catalogue of cometary orbits by Yamamoto [6] extending to 1936 only. As there are only 412 parabolical, hyperbolical and long-periodic orbits (in the sense quoted above) compared with the number 466 in the more recent catalogue by Baldet and de Obaldia, the quantity $N'_R\Delta^{-2}$ has to be multiplied by a factor $k = 0.884$ and comes out

$$N'_R\Delta^{-2} = 185. \tag{28}$$

Accordingly, two random approaches within 0.1 astronomical unit may be expected, which is just the number actually observed. A sample distribution of minimum distances computed by means of (26) is given in the second column of Table VI. For the sake of comparison the actual distances of the three nearest orbits have been extrapolated according to Table X by Guigay [1]; they are given in the third column of the Table together with the designation of the comets in question. It is seen that the agreement of the two distributions is as good as possible and does not require any additional assumption as to the common origin of the bodies.

Table VI

i	Δ_c	Δ_o	Comet
1	0.052	0.074	1877 III
2	0.090	0.079	1932 V
3	0.116	0.125	1825 II

However, the problem investigated by Guigay is a little different from that treated here: instead of the approaches to a given point in the space Guigay examines the approaches to a section of the orbit of Comet 1862 III delimited by true anomalies $v_1 = -47°$ and $v_2 = -67°$, i. e. by $R_1 = 1.140$ and $R_2 = 1.372$. In this case the probability of a random approach of another orbit will be increased by a factor ε, where

$$\varepsilon = 1 + \frac{2}{\pi\Delta}\int_0^{\frac{\pi}{2}} \sin^2\vartheta \, d\vartheta \int_{R_1}^{R_2} \sqrt{\frac{r}{r-q}}\, dr =$$

$$= 1 + \frac{1}{2\Delta}\left(R_2\sqrt{1-\frac{q}{R_2}} - R_1\sqrt{1-\frac{q}{R_1}} + \right.$$

$$\left. + \; q \log_e \frac{\sqrt{R_2} + \sqrt{R_2 - q}}{\sqrt{R_1} + \sqrt{R_1 - q}}\right) \qquad (29)$$

It must be emphasized that the formula (29) is only an approximative one. It holds good for a relatively short orbital arc, in which the curvature is unessential and the probabilities of an approach in different points of the section may be substituted by the mean value putting

$$N'_{12} = \frac{\Delta^2}{2(R_2 - R_1)}\int_{R_1}^{R_2} \frac{n_R + R\frac{dn_R}{dR}}{R^2}\, dR \qquad (30)$$

Furthermore, it is based on the simplyfying assumption that the directions of the motions of comets are distributed isotropically. In our particular case,

$$\varepsilon = 1 + 0.244\,\Delta^{-1}$$
$$N' = 173k\Delta^2 = 153\Delta^2 \qquad (31)$$

Hence the expected number of orbits N'' passing within a distance Δ from the given section of the orbit of Comet 1862 III will be

$$N'' = \varepsilon N' = 153\,\Delta^2 + 37.3\,\Delta \qquad (32)$$

E. g. for $\Delta = 0.05$ we obtain $N'' = 2.2$, for $\Delta = 0.10 - N'' = 5.3$. Again a sample distribution may be derived by means of a formula analogous to (26), i. e. by putting $N'' = \frac{2i-1}{2}$ and solving (32) for Δ. The results of these computations are summarized in Table VII.

Table VII

i	Δ_c	Δ_o	Comet
1	0.013	0.022	1825 II
2	0.035	0.045	1826 V
3	0.055	0.060	1877 III
4	0.072	0.071	1932 V

It is seen that our fundamental conclusion as to the fortuitous character of the phenomenon remains unaltered by extending the investigation to the whole indicated orbital arc. The distribution of distances of the cometary orbits nearest to that of Comet 1862 III coincides well with the expected random distribution (the distances being generally even somewhat greater than expected) and no physical connexion of the comets needs to be inferred.

5. *The Grouping of Meteor Orbits in Guigay's Problem*

In connection with the distribution of the orbits of Perseids only few remarks are to be added. As it is seen from Figure 29 of the quoted paper by Guigay, and has already been pointed out by Wright and Whipple [7], the approaches of the Perseid orbits to the point of the hypothetical collision are not too striking. Moreover, if the orbits of the shower meteors at different dates are deduced assuming a priori the apparent radiant moving at a constant speed along a great circle (which is commonly being done in practice), a regular distribution of the orbits must result. If the limiting orbits approach anywhere to a little distance from one another, also all intermediate orbits must, as a matter of fact, pass through the region of the limiting orbits' approach. A similar point of view, denying the significance of this grouping, was expressed previously by Ahnert [3].

6. *Conclusions*

From all what has been said above it is concluded that the approach of five cometary orbits to the orbit of the parent comet of the Perseid meteor stream may not be considered as an evidence of their common origin. The dynamical conditions of the hypothetical collision are entirely inadmissible and the situation of the orbits corresponds to a random distribution. The phenomenon found by Guigay does not support the opinion that the stream of Perseids was formed by a collision at which the five comets concerned have separated from a common parental body.

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ЛЮБОР КРЕСАК

ЗАМЕТКА К ГИПОТЕЗЕ О РОЗНИКНОВЕНИИ МЕТЕОРНОГО ПОТОКА ПЕРСЕИД СРАЖЕНИЕМ

В своей монографии о метеорном потоке Персеид высказал Гиге (Guigay) догадку, что поток возник сражением матерской кометы 1962 III с иным телом, причем одновременно образовалось пять известных самостоятельных комет 1825 II, 1826 V, 1877 III, 1909 I и 1932 V [1]. К этому предположению вело его то обстоятельство, что все указанные кометы проходят в бросающейся в глаза близости точки, в которой сближаются орбиты Персеид, наблюдаемых в различных ночах.

Новейшие работы о эволюции метеорных потоков показали, что энормная широта потока Персеид может объясняться более долгим влиянием дифференциальных возмущений и вследствие чего, стало быть, нет надобности предполагать большие относительные скорости при отделении метеоров от матерской кометы; наоборот, эти скорости трудно допустить. Причины особого распределения орбит Персеид и группы комет, между которыми, по утверждению Гиге, существует определенная зависимость, не были до сих пор объяснены. В настоящей работе доказывается, что в связи с этим вопросом речь может идти лишь о случайной группировке без какой-либо физической зависимости.

Относительные скорости пяти комет по отношению к комете 1862 III в местах самого большего сближения орбит значительны — од 35 км/сек до 85 км/сек. Взрыв кометного ядра, который бы привел к таким скоростям (в 3—4 порядка большим чем в известных случаях деления комет), трудно себе представить. Если бы мы и допустили эту возможность, получается здесь невязка в распределении больших полуосей, упомянутых комет ($0{,}00 < 1/a < 0{,}04$) и каждое иное предположение, при помощи которого бы мы пытались это распределение объяснить, привело бы к неприемлемым величинам для массы первоначальной кометы и интензивности взрыва. Большие относительные скорости нельзя объяснить и возмущающим влиянием планет после распада; все орбиты являются сравнительно мало возмущаемыми. Решение вопроса, может-ли распределение кометных орбит являться случайным, должно быть основано на расчете вероятностей. Поскольку вопрос случайного приближения кометных орбит к данной точке в солнечной системе имеет более широкое значение, в настоящей работе приводится общее решение. Расчет основан на распределении орбит по каталогу Бальде (Baldet) и Обальдиа (Obaldia) [5] с исключением короткопериодических комет. Так как прямые апсид в этом случае распределены изотропически, вероятное множество орбит, проходящих на расстоянии меньшем чем Δ от данной точки, зависит исключительно от гелиоцентрического расстояния точки R. Внутри сферы радиуса R, для $\Delta \ll R$ средняя вероятность случайного приближения орбита к точке определяется уравнением (6); ожидаемое множество приближений N_R для N_R комет с перигелийным расстоянием $q < R$ очень приближенно может быть дано выражением (15). Для точки, находящейся на гелиоцентрическом расстоянии R, ожидаемое множество приближений N_R определяется выражением (21). Отдельные величины, необходимые для вычисления, приведены в таблице IV и также на рис. 3 и 4. Таблица V и рис. 5 изображают типичное распределение расстояний пяти самых близких орбит в различных расстояниях от солнца; они вычислены по формуле (26).

Применение для случая комет, — на взгляд якобы связанных с потоком Персеид, приводит к однозначному заключению, что зависимость только кажущаяся и сближение орбит в пространстве случайное. Нагляднее всего показывают это таблицы VI и VII, на которых изображено истинное распределение расстояний самых близких орбит от места гипотетического сражения, респ. от 20° дуги орбиты кометы 1862 III, в сравнении с ожидаемым распределением. Согласие наблюдения и расчетов сверх ожидания хорошо: в точке предполагаемого сражения нет никакого сгущения орбит.

Взаимное сближение метеорных орбит также не решающее для теории возникновения. Как указал Райт (Wright) и Уиппл (Whipple) в действительности здесь не идет речь о пересечении орбит; также в точке самого тесного приближения рассеяние еще значительнее. При обычном способе обработки материала, когда наперед предполагается равномерное движение мнимого радианта по главной окруженности, результат Гигея не удивителен. Достаточно, чтобы взаимно приблизились крайние орбиты (в начале и в конце деятельности потока) и остальные орбиты должны также проходить в области самого тесного приближения этих двух орбит.

Из приведенной работы вытекали заключения, что явление описанное Гигеом не доказывает, но и не противоречит догадке о возникновении Персеид сражением и догадке об общем происхождении комет 1925 II, 1826 V, 1862 III, 1877 VI, 1909 I и 1932 V. Относительные скорости этих комет в местах самого тесного сближения орбит ведут к совершенно неприемлемым условиям сражения, респ. взрыва. Само положение орбит соответствует случайному распределению и не указывает никакой физической зависимости.

ĽUBOR KRESÁK

K HYPOTÉZE O VZNIKU METEORICKÉHO ROJA PERZEÍD ZRÁŽKOU

Veľký rozptyl stálych meteorických rojov, ktorých šírka už v blízkosti zemskej dráhy dosahuje niekoľko desatín astronomickej jednotky, možno vysvetliť dvoma zásadne odlišnými spôsobmi:

1. Ak pripustíme dostatočne vysoký vek roja, dajú sa hlavné pozorované znaky jeho stavby (trvanie činnosti, ostrosť maxima, rozptyl radiantov) pripísať stupňu postupného vývoja. Podľa tejto predstavy, ktorá dobre vyhovuje Whipplovej a Dubiagovej domnienke o pomalom vytváraní roja unikaním meteorov z okruhu kometárneho jadra, rozptyl roja je výsledkom dlhého diferenciálneho pôsobenia planetárnych porúch, prípadne v súčinnosti s tlakom žiarenia. Nápadné rozdiely v tvare jednotlivých známych rojov sú predovšetkým príznakom rôzneho veku a nie rôzneho spôsobu vzniku.

2. Ak predpokladáme, že sa meteory z väčšej časti rozptýlia hneď pri vzniku roja a že teda nie vývoj, ale spôsob vzniku určuje jeho neskoršiu stavbu, treba rátať s odlišným pôvodom jednotlivých typov rojov. Takýto názor vedie vo svojich dôsledkoch k nevyhnutnosti katastrofického vzniku (zrážky dvoch telies alebo prudkého výbuchu), pri ktorom meteory opúšťajú jadro vysokými relatívnymi rýchlosťami.

Názor na zásadnú otázku, či charakteristické znaky stavby meteorických rojov určuje vznik alebo vývoj, nie je dosiaľ jednotný. Možno povedať, že väčšina autorov vychádza z logickejšieho stanoviska [1] a že na jeho základe sa už dosiahli i určité úspechy vo výklade pozorovaných zjavov, napr. v prácach Whippla, Hamida, Ahnertovej alebo Plavca. Najnovšia teória druhej skupiny pochádza od Guigaya, ktorý ju aplikoval priamo na vznik meteorického roja Perzeíd.

Vo svojej obsiahlej monografii o tomto roji upozornil Guigay na to, že dráhy Perzeíd, pozorovaných v rôznych nociach, pretínajú sa v priestore alebo aspoň nápadne zbližujú v bode o heliocentrických súradniciach $l = 29°$, $b = +65°$, $r = 1{,}22$ astr. jednotky. Priesečník dráh pokladá Guigay — podobne ako Baldet v prípade Andromedíd — za miesto náhleho vzniku roja zrážkou dvoch telies. Aby tieto telesá identifikoval, vyhľadal podľa elementov všetky známe kométy, ktoré sa k bodu môžu priblížiť, a zisťoval minimálne vzdialenosti ich dráh od dráhy materskej kométy 1862 III. Skutočne sa mu podarilo nájsť päť prípadov nápadného priblíženia, a to u komét 1825 II, 1826 V, 1877 III, 1909 I a 1932 V. Všetky sa v skúmanej oblasti môžu priblížiť ku dráhe kométy 1862 III až na 0,02—0,07 astr. jednotky čiže na vzdialenosť o jeden rád menšiu, ako je dnešná šírka roja v perihéliu. Guigay preto vyslovil domnienku, že nielen kométa 1862 III, ale i ostatných päť má spoločný pôvod s Perzeidami a že vznikli rozdelením jedinej väčšej kométy pri zrážke s iným telesom. Hoci práce iných autorov o vývoji meteorických rojov ukázali, že enormná šírka Perzeíd sa dá ľahko vysvetliť i dlhším pôsobením porúch na pôvodne koncentrovaný roj, zjav opísaný Guigayom nebol dosiaľ uspokojivo vysvetlený. Jeho nové podrobné preskúmanie bolo potrebné nielen pre vyjasnenie otázky vzniku Perzeíd, ale predovšetkým s ohľadom na možné zovšeobecnenie na iné meteorické roje.

Doplnenie Guigayovej teórie — ktorá vychádza z čisto geometrického hľadiska — po stránke dynamickej vedie k celkom neprijateľným záverom. Výpočet relatívnych rýchlostí piatich komét voči kométe 1862 III v miestach najväčšieho priblíženia dráh dáva výsledky od 35 km/s do 85 km/s, teda hodnoty o tri—štyri rády vyššie, ako boli dosiaľ pozorované pri delení kometárnych jadier. (Samé Perzeidy majú podľa fotografických meraní rozptyl rýchlostí najviac ± 1 km/s, z čoho do smeru normály pripadá zhruba 1%.) Výbuch, ktorý by viedol k takým vysokým rýchlostiam, je ťažko predstaviteľný; tým viac, ak sa pri ňom majú vytvoriť podružné kompaktné kometárne jadrá! I keby sme takúto možnosť chceli pripustiť, odporuje jej rozdelenie veľkých polosí. Pri izotropickom výbuchu rýchlosťou 60 km/s by iba pre 0,4% úlomkov platila nerovnosť $0{,}00 < 1/a < 0{,}04$, zatiaľ čo v skutočnosti polosi všetkých šiestich komét, uvádzaných Guigayom do súvislosti s rojom Perzeíd, ležia v týchto hraniciach. Okrem toho je v relatívne krátkom časovom intervale pravdepodobnosť objavu kométy nepriamo úmerná dobe obehu. Pozorované rozdelenie polosí teda ani zďaleka nezodpovedá očakávaniu a na obhajobu Guigayovej domnienky by sme museli zaviesť veľmi násilné predpoklady, že 1. materská kométa roja bola pôvodne o veľa rádov hmotnejšia ako normálne kométy, 2. úlomky o kratších obežných dobách medzičasom všetky zanikli. Vysoké relatívne rýchlosti sa nedajú vysvetliť ani poruchovým pôsobením planét v čase po rozpade: všetky dráhy sú pomerne málo rušené a výsledný účinok porúch by tak či tak bol celkom iný.

Rozhodnutie, či zoskupenie šiestich kometárnych dráh, na ktoré upozornil Guigay, môže byť náhodné, treba založiť na kritériách počtu pravdepodobnosti. Keďže otázka náhodného priblíženia kometárnych dráh má širší význam, v práci sa rieši v zovšeobecnenom tvare. Pri skutočnom priestorovom usporiadaní dráh krátkoperiodických a dlhoperiodických komét je v prvom prípade pravdepodobnosť náhodného priblíženia dvoch dráh funkciou ekliptikálnej šírky a heliocentrickej vzdialenosti, v druhom prípade, kde priamky apsíd sú orientované približne izotropicky, iba funkciou heliocentrickej vzdialenosti. V Guigayovom špeciálnom prípade, kde miesto hypotetickej zrážky leží vo vysokej šírke (+65°), stačí uvažovať iba druhú závislosť pri súčasnom vylúčení vyslovene krátkoperiodických dráh a zanedbaní elipticity dlhoperiodických dráh. Potrebné štatistické dáta možno vybrať z generálneho katalógu Baldeta a de Obaldie alebo — pre jednotnosť s Guigayovým materiálom — zo staršieho katalógu Yamamotovho. Vzorce pre výpočet pravdepodobností náhodného priblíženia dráh sú odvodené v treťom odseku práce; konečný výsledok je znázornený na obrázku 5, ktorý ukazuje očakávané rozdelenie vzdialeností najbližších piatich dráh od ľubovoľne zvoleného bodu v danej heliocentrickej vzdialenosti.

Aplikácia na prípad komét, zdanlivo súvisiacich s rojom Perzeíd, vedie k jednoznačnému záveru, že priblíženie dráh je v tomto prípade celkom náhodné. Najzreteľnejšie to ukazujú tabuľky VI a VII, v ktorých je skutočné rozdelenie vzdialeností najbližších dráh od miesta hypotetickej zrážky, resp. od 20° oblúka dráhy kométy 1862 III, porovnané s očakávaným náhodným rozdelením. Súhlas pozorovania a výpočtu je nad očakávanie dobrý: v bode predpokladanej zrážky sa nevyskytuje nijaké zhustenie kometárnych dráh.

Ani vzájomné priblíženie meteorických dráh nie je rozhodujúce pre teóriu vzniku. Ako už upozornili Wright a Whipple na základe analýzy fotografického materiálu, nejde tu v skutočnosti o pretínanie dráh a aj v bode najväčšieho priblíženia je rozptyl ešte stále značný. Pri bežnom spôsobe spracovania, keď sa vopred predpokladá rovnomerný pohyb zdanlivého radiantu po hlavnej kružnici, Guigayov výsledok nie je nijako prekvapujúci. Stačí totiž, aby sa k sebe približovali krajné dráhy (na začiatku a na konci činnosti roja; pri malom vzájomnom sklone dráh a predpokladanej rovnakej obežnej dobe k tomu dochádza veľmi ľahko) a ostatné meteory musia potom tiež prechádzať oblasťou najväčšieho priblíženia týchto dvoch dráh.

Dochádzame teda k záveru, že zjav opísaný Guigayom nedokazuje, ba ani nepodporuje domnienku o vzniku Perzeíd zrážkou, ani domnienku o spoločnom pôvode komét 1825 II, 1826 V, 1862 III, 1877 III, 1909 I a 1932 V. Relatívne rýchlosti týchto komét v miestach najväčšieho priblíženia ich dráh vedú k celkom neprijateľným podmienkam zrážky, resp. výbuchu. Sama poloha dráh zodpovedá náhodnému usporiadaniu a nenaznačuje nijakú fyzikálnu súvislosť.