On the influence of the corpuscular sputtering on the motion of dust particle

J. Klačka[1] **and I. Kapišinský**[2]

[1] *Department of Astronomy and Astrophysics*
Faculty for Mathematics and Physics
Comenius University
Mlynská dolina
842 15 Bratislava
Czech and Slovak Federal Republic

[2] *Astronomical Institute of the Slovak Academy of Sciences*
059 60 Tatranská Lomnica
Czech and Slovak Federal Republic

Received: February 20, 1992

Abstract. The influence of the corpuscular sputtering on the motion of interplanetary dust particle is investigated. Simultaneous action of the Poynting-Robertson effect, corpuscular drag and corpuscular sputtering is taken into account. Analytical solutions of the basic differential equations are presented for circular orbits.

Key words: interplanetary medium: dust

1. Introduction

In our previous paper (Klačka 1991) we have discussed the influence of the impact erosion mechanism on the motion of dust particle in the Solar System. We have shown that the impact erosion is of smaller importance in comparison to the corpuscular sputtering for particles of radii $s < 50\ \mu m$ (see also Dohnanyi 1978; Kapišinský 1984, 1987, 1990). Since greater part of interplanetary dust particles in the zodiacal cloud or in asteroidal dust bands are of radii $10\ \mu m < s < 50\ \mu m$, it is reasonable to investigate the influence of the corpuscular sputtering on the motion of dust particles.

Contrib. Astron. Obs. Skalnaté Pleso **22**, (1992), 205– 208.

2. Solution of the problem

For the purpose of simplicity we shall deal with circular orbits. Our problem can be analytically solved in this case. Some interesting results can be obtained by taking into account simultaneous action of the corpuscular sputtering and the impact erosion effects together, so we consider also the impact erosion mechanism for the beginning.

According to our previous paper, concerning the mechanism of impact erosion (Klačka 1991), the following equations for $e = 0$ hold:

$$\frac{ds}{dt} = - \frac{\varepsilon^D}{a^2} s^{3/2} - \frac{K}{a^2} , \tag{1}$$

$$\frac{da}{dt} = - \frac{\xi}{as} , \tag{2}$$

where

$$\xi = 9 \times 10^{-8} / \varrho , \tag{3}$$

and a is measured in astronomical units, s in cm, ϱ in $g\ cm^{-3}$ (see Equations (7a), (7c) and (10) in quoted paper).

Equation (2) yields

$$\frac{d}{dt} \left(\frac{1}{2} a^2 \right) = - \frac{\xi}{s} , \tag{4}$$

and we have

$$\frac{1}{2} \left(a^2 - a_0^2 \right) = - \int_{t_0}^{t} \frac{\xi}{s} \, dt = \int_{s_0}^{s} \frac{\xi}{s} \left(\frac{ds}{dt} \right)^{-1} ds . \tag{5}$$

Substitution of Equation (1) into the last equation yields:

$$\frac{1}{2} \left(a^2 - a_0^2 \right) = \int_{s_0}^{s} \frac{\xi}{s} a^2 \left(\varepsilon^D s^{3/2} + K \right)^{-1} ds . \tag{6}$$

There is $s = s(a)$ or $a = a(s)$ in Equation (6). We can find the dependence of s on a by the differentiation of Equation (6) with respect to s:

$$\frac{da}{ds} = \frac{\xi}{s} a \left(\varepsilon^D s^{3/2} + K \right)^{-1} \tag{7}$$

and finally

$$ln \left(\frac{a}{a_0} \right) = \xi \int_{s_0}^{s} \frac{ds}{s \left(\varepsilon^D s^{3/2} + K \right)} . \tag{8}$$

Integral in Equation (8) can be solved analytically. One can obtain

$$a = a_0 \left(\frac{s}{s_0} \right)^{\xi / K} \left(\frac{\varepsilon^D s_0^{3/2} + K}{\varepsilon^D s^{3/2} + K} \right)^{(2/3) \xi / K} . \tag{9}$$

Substitution of the last equation into Equation (1) yields

$$\frac{ds}{dt} = - \Phi \left(\varepsilon^D \, s^{3/2} + K\right)^{1+(4/3)\, \xi/K} s^{-2\, \xi/K} , \tag{10}$$

where constant Φ is defined by formula

$$\Phi = \frac{1}{a_0^2} \left\{ \frac{s_0^2}{\left(\varepsilon^D \, s_0^{3/2} + K\right)^{4/3}} \right\}^{\xi/K} . \tag{11}$$

Using the fact that $\xi/K = (45/2)\, \varrho$,

$$- \Phi \int_{t_0}^{t} dt = \int_{s_0}^{s} \frac{s^{45/\varrho}}{\left(\varepsilon^D \, s^{3/2} + K\right)^{1+30/\varrho}} \, ds . \tag{12}$$

We can use substitution $s = x^2$ in the last integral. It is the integral of binomial expression and it leads to elementary functions only in some special cases (e. g., $\varrho = 1,\ 3$) - see, e. g., Ryshik and Gradstein (1957), p. 67.

We shall not take into account the impact erosion in the following calculations (formally: $\varepsilon^D = 0$). Integral (12) can be easily solved in this case and finally it gives

$$s = s_0 \, \Lambda^\beta , \tag{13}$$

where

$$\Lambda = \left(1 - \xi \, \frac{2 + \beta}{s_0 \, a_0^2} \, t\right)^{1/(2+\beta)} \tag{14}$$

and

$$\beta = K/\xi = (2/45)\, \varrho . \tag{15}$$

Using also Equation (9), it yields

$$a = a_0 \, \Lambda . \tag{16}$$

3. Discussion

Equations (13) and (16) hold for the case

$$s > 5.7 \times 10^{-5}/\varrho . \tag{17}$$

Dust particles are expelled from the Solar System in the opposite case (see Equation (4) in Klačka 1991). However, the corpuscular sputtering has such a small destructive character, that this situation does not occur for particles of the initial radius $s_0 > 1\ \mu m$ in the real situation - particles fall on the Solar surface or are evaporated in the vicinity of the Sun.

ϱ $[gcm^{-3}]$	a_0 $[AU]$	t/t_{PRCD}	s/s_0
3.0	3.0	0.940	0.736
3.0	1.0	0.951	0.852
1.0	3.0	0.979	0.903
1.0	1.0	0.983	0.948

Table 1. Quantities characterizing the importance of the corpuscular sputtering. ($a_0 \longrightarrow a = 0.3AU$)

Some other interesting results are presented in Table 1. This table characterizes (as an example) the influence of the corpuscular sputtering on the motion of dust particles. Poynting-Robertson and corpuscular drags, and corpuscular sputtering are taken into account simultaneously (see Equations (13) and (16)). Time t has elapsed during particle's motion from the initial solar distance a_0 to $a = 0.3$ AU; particle's initial radius s_0 has decreased to the value s during this motion. Time t_{PRCD} corresponds to the situation when only Poynting-Robertson and corpuscular drags are taken into account. All quantities presented in Table 1 do not depend on the initial value of s_0:

$$t = \frac{s_0 \, a_0^2}{2+\beta} \frac{1}{\xi} \left[1 - \left(\frac{a}{a_0} \right)^{2+\beta} \right] , \tag{18}$$

$$t_{PRCD} = \frac{s_0 \, a_0^2}{2 \, \xi} \left[1 - \left(\frac{a}{a_0} \right)^{2} \right] . \tag{19}$$

References

Dohnanyi, J. A.: 1978, in *Cosmic Dust*, ed(s). J. A. M. McDonnell, J. Wiley and Sons, Chichester, 527

Kapišinský, I.: 1984, *Bull. Astron.Inst. Czechosl.* **35**, 300

Kapišinský, I.: 1987, *Bull. Astron.Inst. Czechosl.* **38**, 7

Kapišinský, I.: 1990, *Contrib. Astron. Obs. Skalnaté Pleso* **42**, 183

Klačka, J.: 1991, *Bull. Astron.Inst. Czechosl.* **42**, 379

Ryshik, I. M. and Gradstein, I. S.: 1957, *Tables of Series, Products and Integrals*, VEB Deutscher Verlag der Wissenschaften, Berlin