Applicability of meteor radiant determination methods depending on orbit type

II. Low-eccentric orbits

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Abstract. All known parent bodies of meteor showers belong to bodies moving in high-eccentricity orbits ($e \geq 0.5$). Recently, asteroids in low-eccentricity orbits ($e < 0.5$) approaching the Earth's orbit, were suggested as another population of possible parent bodies of meteor streams. This paper deals with the problem of calculation of meteor radiants connected with the bodies in low-eccentricity orbits from the point of view of optimal results depending on the method applied. The paper is a continuation of our previous analysis of high-eccentricity orbits (Svoreň et al, 1993). Some additional methods resulting from mathematical modelling are presented and discussed together with Porter's, Steel-Baggaley's and Hasegawa's methods.

In order to be able to compare how suitable the application of the individual radiant determination methods is, it is necessary to determine the accuracy with which they approximate real meteor orbits. To verify the accuracy with which the orbit of a meteoroid with at least one node at 1 AU fits the original orbit of the parent body, the Southworth-Hawkins D-criterion (Southworth and Hawkins, 1963) was applied. $D \leq 0.1$ indicates a very good fit of orbits, $0.1 < D \leq 0.2$ is considered for a good fit and $D > 0.2$ means that the fit is rather poor and the change of orbit unrealistic.

The optimal method, i.e. the one which results in the smallest D values for the population of low-eccentricity orbits, is that of adjusting the orbit by varying both the eccentricity and perihelion distance.

A comparison of theoretical radiants obtained by various methods was made for typical representatives from each group of the NEA (near-Earth asteroids) objects.

Key words: asteroids – meteor streams

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1. Introduction

The paper is a continuation of our previous analysis of high-eccentric orbits (Svoreň et al, 1993) further referred to as Paper 1.

The methods of radiant calculation as well as mathematical formulas used are not repeated here. These are given in the appendix of Paper 1. Except for the well-known standard methods (Porter, Steel-Baggaley and Hasegawa) some additional methods resulting from mathematical modelling are studied in both papers.

A brief discussion of the results was already presented at the IAU Symp. 160 in Belgirate and accepted in the Planetary and Space Science journal. The present paper provides full information about the analysis, including the plots for chosen discrete values of elements of low-eccentricity orbits and recommends the most appropriate method of meteor radiant calculation pertinent to the given type of orbit.

2. Modelling

The present analysis deals only with the low-eccentricity orbits. It also states the examined ranges for the individual orbital elements. The model calculations were based on discrete values of q (0.1-1.2 AU, step 0.1 AU), e (0.05, 0.20, 0.35), ω (0°-360°, step 15°), i (5°-80°and 100°-175°, step 15°). Change in the ascending node Ω was not taken into account, because if the eccentricity of the Earth's orbit is neglected the result is independent of Ω.

As we have to take into account the closest approaches along both pre- and post-perihelion arcs of the orbits of the parent bodies (two different streams can be associated), the argument of perihelion was investigated in the interval from 0°to 360°. It can be shown that the result is symmetrical around the value $\omega = 180°$ (i.e. the values for ω and $360° - \omega$ are the same). As this is valid for all the methods, only the results in the interval of ω from 0°to 180°are shown in the plots.

It is very advantageous to distinguish the arcs of the original orbit with respect to the ascending node. In the both papers *the arc of the ascending node* is the part of the original orbit between the perihelion and aphelion including the ascending node; *the arc of the descending node* is the second half of the orbit. Transformation from the arcs thus defined to the preperihelion or postperihelion arcs depends on the position of the perihelion with regard to the ecliptic plane (see Table 8 of Paper 1).

The results of the methods of adjustment of the orbits by changes in the orbital planes of the parent bodies are shown in Fig. 1-4. The circles indicate the degree of fit the method provides. Filled circles indicate that the method provides a very good fit ($D \leq 0.1$), the open circles indicate that the method

	ascending arc												
	180°	165°	150°	135°	120°	105°	90°	75°	60°	45°	30°	15°	0°
	180°	195°	210°	225°	240°	255°	270°	285°	300°	315°	330°	345°	360°
1.2	○○○	···	···	···	···	···	···	···	···	···	···	···	···
1.1	●●●	○○○	○○○	○○○	○○·	○··	○··	○··	○··	○··	○··	○··	○··
1.0	×××	●●●	●●●	●●●	●●○	●○○	●○·	●··	●··	●··	●··	●··	×××
0.9	●●●	●●●	●●●	●●●	●●●	●●●	●●○	●○·	●○·	●○·	●··	●··	●··
0.8	○○○	○○○	○○○	○○○	○○●	○●●	○●●	○●○	○●○	○●·	○○·	○○·	○○·
0.7	···	···	···	···	··○	·○○	·○●	·●●	·●●	·●○	·●○	·●·	·●·
0.6	···	···	···	···	···	···	··○	·○●	·○●	·○●	·●●	·●○	·●○
0.5	···	···	···	···	···	···	···	··○	··○	··●	·○●	·○●	·○●
0.4	···	···	···	···	···	···	···	···	···	··○	··○	··●	··●
0.3	···	···	···	···	···	···	···	···	···	···	···	··○	··○
0.2	···	···	···	···	···	···	···	···	···	···	···	···	···
0.1	···	···	···	···	···	···	···	···	···	···	···	···	···
	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
	360°	345°	330°	315°	300°	285°	270°	255°	240°	225°	210°	195°	180°
	desending arc												

Figure 1. $q = q(\omega)$, Q - q-adjustment method.

	ascending arc												
	180°	165°	150°	135°	120°	105°	90°	75°	60°	45°	30°	15°	0°
	180°	195°	210°	225°	240°	255°	270°	285°	300°	315°	330°	345°	360°
1.2	···	···	···	···	···	···	···	···	···	···	···	···	···
1.1	···	···	···	···	···	···	···	···	···	···	···	···	···
1.0	×××	●○·	●○·	●○·	●○·	●○·	●○·	●○·	●○·	●○·	●○·	●○·	×××
0.9	···	···	···	··○	○●●	○●○	●●·	●○·	●○·	●○·	●○·	●○·	●○·
0.8	···	···	··	···	···	·○●	○●●	○●○	○●○	●●·	●●·	●●·	●●·
0.7	···	···	···	···	···	···	··●	·○●	○●●	○●○	○●○	○●○	○●○
0.6	···	···	···	···	···	···	···	··○	·○●	·○●	·●●	·●●	○●●
0.5	···	···	···	···	···	···	···	···	··○	··●	·○●	·○●	·○●
0.4	···	···	···	···	···	···	···	···	···	··○	··○	··●	··●
0.3	···	···	···	···	···	···	···	···	···	···	···	···	··○
0.2	···	···	···	···	···	···	···	···	···	···	···	···	···
0.1	···	···	···	···	···	···	···	···	···	···	···	···	···
	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
	360°	345°	330°	315°	300°	285°	270°	255°	240°	225°	210°	195°	180°
	desending arc												

Figure 2. $q = q(\omega)$, E - method of varying the eccentricity.

	ascending arc												
	180°	165°	150°	135°	120°	105°	90°	75°	60°	45°	30°	15°	0°
	180°	195°	210°	225°	240°	255°	270°	285°	300°	315°	330°	345°	360°
1.2	···	···	···	···	···	···	···	···	···	···	···	···	···
1.1	···	···	···	···	···	···	···	···	···	···	···	···	···
1.0	×××	●●●	●○○	●○·	●○·	●··	●··	●··	●○·	●○·	●○○	●●●	×××
0.9	···	·○·	·○○	·●●	·●●	·●○	·●·	·○·	·○·	···	···	···	···
0.8	···	···	···	·○○	·○○	·●●	·●●	·●○	·●·	·○·	·○·	···	···
0.7	·○·	·○·	···	···	···	··○	·○●	·○●	·●○	·●·	·●·	·●·	·○·
0.6	···	···	···	···	···	···	··○	··○	··●	··●	··○	···	···
0.5	··○	···	···	···	···	···	···	···	···	··○	··●	··●	··○
0.4	···	···	···	···	···	···	···	···	···	···	···	···	···
0.3	···	···	···	···	···	···	···	···	···	···	···	···	···
0.2	···	···	···	···	···	···	···	···	···	···	···	···	···
0.1	···	···	···	···	···	···	···	···	···	···	···	···	···
	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
	360°	345°	330°	315°	300°	285°	270°	255°	240°	225°	210°	195°	180°
	desending arc												

Figure 3. $q = q(\omega)$, W - method of varying the argument of perihelion.

	ascending arc												
	180°	165°	150°	135°	120°	105°	90°	75°	60°	45°	30°	15°	0°
	180°	195°	210°	225°	240°	255°	270°	285°	300°	315°	330°	345°	360°
1.2	○○○	···	···	···	···	○··	○··	○··	○··	○··	○··	○··	○··
1.1	●●●	○○○	○○○	○○○	○○·	○○·	○··	○··	●··	●··	●··	●··	●··
1.0	×××	●●●	●●●	●●●	●●○	●○○	●○·	●○·	●○·	●○·	●○·	●○·	×××
0.9	●●●	●●●	●●●	●●●	●●●	●●●	●●○	●●○	●○·	●○·	●○·	●○·	●○·
0.8	○○○	○○○	○○○	○○○	○○●	○●●	○●●	●●○	●●○	●●○	●●○	●●○	●●○
0.7	···	···	···	···	··○	·○○	○○●	○●●	○●●	○●○	○●○	○●○	○●○
0.6	···	···	···	···	···	··○	·○○	·○●	·○●	○●●	○●●	○●●	○●●
0.5	···	···	···	···	···	···	···	··○	·○●	·○●	·○●	·○●	·○●
0.4	···	···	···	···	···	···	···	···	··○	··○	·○●	·○●	·○●
0.3	···	···	···	···	···	···	···	···	···	··○	··○	··○	··○
0.2	···	···	···	···	···	···	···	···	···	···	···	···	··○
0.1	···	···	···	···	···	···	···	···	···	···	···	···	···
	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
	360°	345°	330°	315°	300°	285°	270°	255°	240°	225°	210°	195°	180°
	desending arc												

Figure 4. $q = q(\omega)$, B - method of varying both the perihelion distance and eccentricity.

ascending arc

	180°	165°	150°	135°	120°	105°	90°	75°	60°	45°	30°	15°	0°
	180°	195°	210°	225°	240°	255°	270°	285°	300°	315°	330°	345°	360°
1.2	···	···	···	···	···	···	···	···	···	···	···	···	···
1.1	···	···	···	···	···	···	···	···	···	···	···	···	···
1.0	×××	●●●	○○○	···	···	···	···	···	···	···	○○○	●●●	×××
0.9	···	···	···	···	···	···	·○·	·●○	·●●	·○●	·○○	···	···
0.8	···	···	···	·○·	·○·	·●○	·●●	·○●	··●	··○	···	···	···
0.7	·○·	·○·	·●·	·●○	·○○	··●	··●	··○	···	···	···	···	·○·
0.6	···	···	··○	··●	··●	··○	··○	···	···	···	···	···	···
0.5	··○	··●	··●	··○	···	···	···	···	···	···	···	··○	··○
0.4	···	···	···	···	···	···	···	···	···	···	···	···	···
0.3	···	···	···	···	···	···	···	···	···	···	···	···	···
0.2	···	···	···	···	···	···	···	···	···	···	···	···	···
0.1	···	···	···	···	···	···	···	···	···	···	···	···	···
	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
	360°	345°	330°	315°	300°	285°	270°	255°	240°	225°	210°	195°	180°

desending arc

Figure 5. $q = q(\omega)$, H - ω-adjustment method, inclination = 20°or 160°.

ascending arc

	180°	165°	150°	135°	120°	105°	90°	75°	60°	45°	30°	15°	0°
	180°	195°	210°	225°	240°	255°	270°	285°	300°	315°	330°	345°	360°
1.2	···	···	···	···	···	···	···	···	···	···	···	···	···
1.1	···	···	···	···	···	···	···	···	···	···	···	···	···
1.0	×××	···	···	···	···	···	···	···	···	···	···	···	×××
0.9	·●●	·●●	·●●	·●●	·●●	·●●	·●●	·●●	·●○	·●○	·●○	·●○	·●●
0.8	·●●	·●●	·●●	·●●	·●●	·●●	·●●	·●●	·●●	·●●	·●●	·●●	·●●
0.7	·●●	·○●	·○●	·○●	·○●	·○●	·○●	·●●	·●●	·●●	·●●	·●●	·●●
0.6	··●	··●	··○	··○	··●	··●	··●	··●	··●	··●	··●	··●	··●
0.5	··●	··○	··○	···	···	···	···	··○	··○	··●	··●	··●	··●
0.4	···	···	···	···	···	···	···	···	···	···	···	···	···
0.3	···	···	···	···	···	···	···	···	···	···	···	···	···
0.2	···	···	···	···	···	···	···	···	···	···	···	···	···
0.1	···	···	···	···	···	···	···	···	···	···	···	···	···
	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
	360°	345°	330°	315°	300°	285°	270°	255°	240°	225°	210°	195°	180°

desending arc

Figure 6. $q = q(\omega)$, A - method of rotation around the line of apsides, inclination = 5°or 175°.

is acceptable ($0.1 < D \leq 0.2$), points indicate that the method is unacceptable, and "×" indicates that the parent body's orbit crosses the Earth's orbit.

The results for the two methods of adjustment of the orbit by changes in angular elements are shown as examples in Fig. 5-6. The plots including all three methods and all chosen inclinations cannot be shown here because of the lack of space. But, the next section summarizes the most convenient methods for the chosen types of orbits.

3. Methods recommended for various types of orbits

The optimal methods with the smallest values of D for given types of orbits are shown in two series of six plots (Figs. 7-18). Maximum D-values in the first and second series are 0.1 and 0.2, respectively. All the plots are symmetrical with respect to $i = 90°$. That is why we have common plots for $i = 5°$ and $i = 175°$, and so on.

In the $q = q(\omega)$ plots, each point is marked with three letters which, from left to right, represent the eccentricity of 0.05, 0.20, 0.35.

The individual letters represent:

Q - q-adjustment method (Hasegawa, 1990),
B - method of variation of both the perihelion distance and eccentricity,
W - method of variation of the argument of perihelion (Steel and Baggaley, 1985),
H - ω-adjustment method (Hasegawa, 1990),
A - method of rotation around the line of apsides,
P - Porter's method of the velocity vector shift (Porter, 1952).

There are a few combinations of elements in which the values of D are the same or very similar to each other for two or more methods. The values of D are considered to be the same if the difference between them is smaller than 0.001. Priority is then given to the less complicated method (in the order Q, E, W, H, A, B, P). This choice is justified by a more simple mathematical elaboration and simultaneously by the same quality of results.

Besides the letters in the plots representing particular methods, the following additional symbols are used:

1 - no method yields $D \leq 0.1$, and a part of the orbit is closer to the Earth's orbit than 0.2 AU,
2 - no method yields $D \leq 0.2$, and a part of the orbit is closer to the Earth's orbit than 0.2 AU,
$\diamond$ - no method yields $D \leq 0.1$, and the whole orbit is far from the Earth's orbit (more than 0.2 AU),
$*$ - no method yields $D \leq 0.2$, and the whole orbit is far from the Earth's orbit (more than 0.2 AU),
× - the cometary orbit crosses the Earth's orbit.

ascending arc

	180° 180°	165° 195°	150° 210°	135° 225°	120° 240°	105° 255°	90° 270°	75° 285°	60° 300°	45° 315°	30° 330°	15° 345°	0° 360°
1.2	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇
1.1	QQQ	111	111	111	111	111	111	111	B11	B11	B11	B11	B11
1.0	×××	QQQ	QQP	QPP	BPP	BPP	BHH	BHH	BHH	WHH	WHH	WHH	×××
0.9	PHH	QAP	QAP	BAP	BAP	BAP	BHH	BHH	BHH	BHH	EHH	EHH	EHH
0.8	1HH	1HA	1HA	1HA	1HA	1HA	1HH	BAH	BAH	BAH	BAH	BBH	BBH
0.7	◇HH	◇HH	◇HH	◇HH	◇HH	◇HH	◇HH	◇PA	◇BA	◇BA	◇BA	◇BA	◇BH
0.6	◇1H	◇1H	◇1H	◇1H	◇1H	◇1H	◇1H	◇1P	◇1A	◇BP	◇BP	◇BP	◇BH
0.5	◇◇H	◇◇H	◇◇H	◇◇H	◇◇H	◇◇H	◇◇H	◇◇P	◇◇P	◇◇P	◇◇B	◇◇B	◇◇B
0.4	◇◇1	◇◇1	◇◇1	◇◇1	◇◇1	◇◇1	◇◇1	◇◇1	◇◇1	◇◇1	◇◇B	◇◇B	◇◇B
0.3	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇
0.2	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇
0.1	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇
	0° 360°	15° 345°	30° 330°	45° 315°	60° 300°	75° 285°	90° 270°	105° 255°	120° 240°	135° 225°	150° 210°	165° 195°	180° 180°

desending arc

Figure 7. $q = q(\omega)$, $D \leq 0.1$, inclination = 5°or 175°.

ascending arc

	180° 180°	165° 195°	150° 210°	135° 225°	120° 240°	105° 255°	90° 270°	75° 285°	60° 300°	45° 315°	30° 330°	15° 345°	0° 360°
1.2	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇
1.1	QQQ	111	111	111	11◇	1◇◇	1◇◇	1◇◇	B◇◇	B◇◇	B11	B11	B11
1.0	×××	QQQ	QQQ	QQQ	BB1	B11	B1◇	B◇◇	B◇◇	W◇◇	W11	WWH	×××
0.9	PQQ	QQQ	QQQ	BBQ	BQB	BBB	BB1	BH◇	BHH	B◇H	E11	E11	E11
0.8	1◇◇	111	111	111	11B	1HQ	1BB	BBH	BBH	BB◇	BB◇	BB◇	BB◇
0.7	◇1◇	◇1◇	◇H◇	◇H◇	◇◇1	◇1H	◇1B	◇BB	◇BB	◇B1	◇B◇	◇B◇	◇B◇
0.6	◇1◇	◇1◇	◇1◇	◇◇H	◇◇H	◇◇1	◇◇1	◇◇B	◇1B	◇BB	◇BB	◇BB	◇BB
0.5	◇◇1	◇◇H	◇◇H	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇1	◇◇B	◇◇B	◇◇B	◇◇B	◇◇B
0.4	◇◇1	◇◇1	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇B	◇◇B	◇◇B
0.3	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇
0.2	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇
0.1	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇
	0° 360°	15° 345°	30° 330°	45° 315°	60° 300°	75° 285°	90° 270°	105° 255°	120° 240°	135° 225°	150° 210°	165° 195°	180° 180°

desending arc

Figure 8. $q = q(\omega)$, $D \leq 0.1$, inclination = 20°or 160°.

ascending arc

	180° 180°	165° 195°	150° 210°	135° 225°	120° 240°	105° 255°	90° 270°	75° 285°	60° 300°	45° 315°	30° 330°	15° 345°	0° 360°
1.2	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄
1.1	QQQ	111	111	111	11⋄	1⋄⋄	1⋄⋄	1⋄⋄	B⋄⋄	B⋄⋄	B⋄⋄	B11	B11
1.0	×××	QQQ	QQQ	QQQ	BB1	B1⋄	B1⋄	B⋄⋄	B⋄⋄	W⋄⋄	W⋄⋄	WWW	×××
0.9	PQQ	QQQ	QQQ	BBQ	BQB	BBB	BB1	BH⋄	BHH	B⋄H	E⋄⋄	E11	E11
0.8	1⋄⋄	111	111	111	11B	1HQ	1BB	BBH	BB⋄	BB⋄	BB⋄	BB⋄	BB⋄
0.7	⋄1⋄	⋄1⋄	⋄H⋄	⋄H⋄	⋄⋄1	⋄1H	⋄1B	⋄BB	⋄BB	⋄B⋄	⋄B⋄	⋄B⋄	⋄B⋄
0.6	⋄1⋄	⋄1⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄H	⋄⋄⋄	⋄⋄1	⋄⋄B	⋄1B	⋄BB	⋄BB	⋄BB	⋄BB
0.5	⋄⋄1	⋄⋄H	⋄⋄H	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄B	⋄⋄B	⋄⋄B	⋄⋄B	⋄⋄B
0.4	⋄⋄1	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄B	⋄⋄B	⋄⋄B
0.3	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄
0.2	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄
0.1	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄
	0° 360°	15° 345°	30° 330°	45° 315°	60° 300°	75° 285°	90° 270°	105° 255°	120° 240°	135° 225°	150° 210°	165° 195°	180° 180°

desending arc

Figure 9. $q = q(\omega)$, $D \leq 0.1$, inclination = 35° or 145°.

ascending arc

	180° 180°	165° 195°	150° 210°	135° 225°	120° 240°	105° 255°	90° 270°	75° 285°	60° 300°	45° 315°	30° 330°	15° 345°	0° 360°
1.2	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄
1.1	QQQ	111	111	111	11⋄	1⋄⋄	1⋄⋄	1⋄⋄	B⋄⋄	B⋄⋄	B⋄⋄	B⋄⋄	B11
1.0	×××	QQQ	QQQ	QQQ	BB1	B1⋄	B1⋄	B⋄⋄	B⋄⋄	W⋄⋄	W⋄⋄	WWW	×××
0.9	PQQ	QQQ	QQQ	BBQ	BQB	BBB	BB1	BH⋄	BHH	B⋄H	E⋄⋄	E11	E11
0.8	1⋄⋄	111	111	111	11B	1HQ	1BB	BBH	BB⋄	BB⋄	BB⋄	BB⋄	BB⋄
0.7	⋄1⋄	⋄1⋄	⋄H⋄	⋄H⋄	⋄⋄1	⋄1H	⋄1B	⋄BB	⋄BB	⋄B⋄	⋄B⋄	⋄B⋄	⋄B⋄
0.6	⋄1⋄	⋄1⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄H	⋄⋄⋄	⋄⋄1	⋄⋄B	⋄1B	⋄BB	⋄BB	⋄BB	⋄BB
0.5	⋄⋄1	⋄⋄H	⋄⋄H	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄B	⋄⋄B	⋄⋄B	⋄⋄B	⋄⋄B
0.4	⋄⋄1	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄B	⋄⋄B	⋄⋄B
0.3	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄
0.2	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄
0.1	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄
	0° 360°	15° 345°	30° 330°	45° 315°	60° 300°	75° 285°	90° 270°	105° 255°	120° 240°	135° 225°	150° 210°	165° 195°	180° 180°

desending arc

Figure 10. $q = q(\omega)$, $D \leq 0.1$, inclination = 50° or 130°.

ascending arc

	180°	165°	150°	135°	120°	105°	90°	75°	60°	45°	30°	15°	0°
	180°	195°	210°	225°	240°	255°	270°	285°	300°	315°	330°	345°	360°
1.2	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇
1.1	QQQ	111	111	111	11◇	1◇◇	1◇◇	1◇◇	B◇◇	B◇◇	B◇◇	B◇◇	B11
1.0	×××	QQQ	QQQ	QQQ	BB1	B1◇	B1◇	B◇◇	B◇◇	W◇◇	W◇◇	WWW	×××
0.9	PQQ	QQQ	QQQ	BBQ	BQB	BBB	BB1	BH◇	BHH	B◇H	E◇◇	E◇◇	E11
0.8	1◇◇	111	111	111	11B	1HQ	1BB	BBH	BB◇	BB◇	BB◇	BB◇	BB◇
0.7	◇1◇	◇◇◇	◇H◇	◇H◇	◇◇1	◇1H	◇1B	◇BB	◇BB	◇B◇	◇B◇	◇B◇	◇B◇
0.6	◇1◇	◇◇◇	◇◇◇	◇◇◇	◇◇H	◇◇◇	◇◇1	◇◇B	◇1B	◇BB	◇BB	◇BB	◇BB
0.5	◇◇1	◇◇H	◇◇H	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇B	◇◇B	◇◇B	◇◇B	◇◇B
0.4	◇◇1	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇B	◇◇B	◇◇B
0.3	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇
0.2	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇
0.1	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇
	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
	360°	345°	330°	315°	300°	285°	270°	255°	240°	225°	210°	195°	180°

desending arc

Figure 11. $q = q(\omega)$, $D \leq 0.1$, inclination = 65°or 115°.

ascending arc

	180°	165°	150°	135°	120°	105°	90°	75°	60°	45°	30°	15°	0°
	180°	195°	210°	225°	240°	255°	270°	285°	300°	315°	330°	345°	360°
1.2	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇
1.1	QQQ	111	111	111	11◇	1◇◇	1◇◇	1◇◇	B◇◇	B◇◇	B◇◇	B◇◇	B11
1.0	×××	QQQ	QQQ	QQQ	BB1	B1◇	B1◇	B◇◇	B◇◇	W◇◇	W◇◇	WWW	×××
0.9	PQQ	QQQ	QQQ	BBQ	BQB	BBB	BB◇	BH◇	BHH	BHH	E◇◇	E◇◇	E11
0.8	1◇◇	111	111	111	1HB	1HQ	1BB	BBH	BB◇	BB◇	BB◇	BB◇	BB◇
0.7	◇1◇	◇◇◇	◇H◇	◇H◇	◇◇1	◇1H	◇1B	◇BB	◇BB	◇B◇	◇B◇	◇B◇	◇B◇
0.6	◇1◇	◇◇◇	◇◇◇	◇◇H	◇◇H	◇◇◇	◇◇1	◇◇B	◇1B	◇BB	◇BB	◇BB	◇BB
0.5	◇◇1	◇◇H	◇◇H	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇B	◇◇B	◇◇B	◇◇B	◇◇B
0.4	◇◇1	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇B	◇◇B	◇◇B
0.3	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇
0.2	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇
0.1	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇
	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
	360°	345°	330°	315°	300°	285°	270°	255°	240°	225°	210°	195°	180°

desending arc

Figure 12. $q = q(\omega)$, $D \leq 0.1$, inclination = 80°or 100°.

ascending arc

	180°	165°	150°	135°	120°	105°	90°	75°	60°	45°	30°	15°	0°
	180°	195°	210°	225°	240°	255°	270°	285°	300°	315°	330°	345°	360°
1.2	QQQ	***	***	***	***	B**	B**	B**	B**	B**	B**	B**	B**
1.1	QQQ	QQQ	QQQ	BBP	BPP	BPP	BPP	B22	B22	B22	B22	B22	B22
1.0	×××	QQQ	QQP	QPP	BPP	BPP	BHH	BHH	BHH	WHH	WHH	WHH	×××
0.9	PHH	QAP	QAP	BAP	BAP	BAP	BHH	BHH	BHH	BHH	EHH	EHH	EHH
0.8	QHH	QHA	BHA	BHA	BHA	BHA	BHH	BAH	BAH	BAH	BAH	BBH	BBH
0.7	*HH	*HH	*HH	*HH	*HH	*HH	BHH	BPA	BBA	BBA	BBA	BBA	BBH
0.6	*2H	*2H	*2H	*2H	*2H	*2H	*BH	*BP	*BA	BBP	BBP	BBP	BBH
0.5	**H	**H	**H	**H	**H	**H	**H	**P	*BP	*BP	*BB	*BB	*BB
0.4	**2	**2	**2	**2	**2	**2	**2	**2	**B	**B	*BB	*BB	*BB
0.3	***	***	***	***	***	***	***	***	***	**B	**B	**B	**B
0.2	***	***	***	***	***	***	***	***	***	***	***	***	**B
0.1	***	***	***	***	***	***	***	***	***	***	***	***	***
	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
	360°	345°	330°	315°	300°	285°	270°	255°	240°	225°	210°	195°	180°

desending arc

Figure 13. $q = q(\omega)$, $D \leq 0.2$, inclination = 5°or 175°.

ascending arc

	180°	165°	150°	135°	120°	105°	90°	75°	60°	45°	30°	15°	0°
	180°	195°	210°	225°	240°	255°	270°	285°	300°	315°	330°	345°	360°
1.2	QQQ	***	***	***	***	B**	B**	B**	B**	B**	B**	B**	B**
1.1	QQQ	QQQ	QQQ	BBB	BB*	BB*	B**	B**	B**	B**	B22	B22	B22
1.0	×××	QQQ	QQQ	QQQ	BBB	BBB	BB*	BB*	BB*	WW*	WWH	WWH	×××
0.9	PQQ	QQQ	QQQ	BBQ	BQB	BBB	BBB	BHH	BHH	BBH	EBH	EB2	EB2
0.8	QQQ	QQQ	BQQ	BBB	BHB	BHQ	BBB	BBH	BBH	BBB	BBB	BBB	BBB
0.7	*W*	*H*	*H*	*HH	*HH	*BH	BBB	BBB	BBB	BBB	BBB	BBB	BBB
0.6	*2*	*2*	*2H	**H	**H	**H	*BB	*BB	*BB	BBB	BBB	BBB	BBB
0.5	**H	**H	**H	**H	***	***	***	**B	*BB	*BB	*BB	*BB	*BB
0.4	**2	**2	***	***	***	***	***	***	**B	**B	*BB	*BB	*BB
0.3	***	***	***	***	***	***	***	***	***	**B	**B	**B	**B
0.2	***	***	***	***	***	***	***	***	***	***	***	***	**B
0.1	***	***	***	***	***	***	***	***	***	***	***	***	***
	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
	360°	345°	330°	315°	300°	285°	270°	255°	240°	225°	210°	195°	180°

desending arc

Figure 14. $q = q(\omega)$, $D \leq 0.2$, inclination = 20°or 160°.

ascending arc

	180°	165°	150°	135°	120°	105°	90°	75°	60°	45°	30°	15°	0°
	180°	195°	210°	225°	240°	255°	270°	285°	300°	315°	330°	345°	360°
1.2	QQQ	***	***	***	***	B**	B**	B**	B**	B**	B**	B**	B**
1.1	QQQ	QQQ	QQQ	BBB	BB*	BB*	B**	B**	B**	B**	B**	B22	B22
1.0	×××	QQQ	QQQ	QQQ	BBB	BBB	BB*	BB*	BB*	WW*	WWW	WWW	×××
0.9	PQQ	QQQ	QQQ	BBQ	BQB	BBB	BBB	BHB	BHH	BBH	EBH	EB2	EB2
0.8	QQQ	QQQ	BQQ	BBB	BBB	BHQ	BBB	BBH	BBH	BBB	BBB	BBB	BBB
0.7	*W*	*W*	*H*	*H*	*HB	*BH	BBB	BBB	BBB	BBB	BBB	BBB	BBB
0.6	*2*	*2*	***	**H	**H	**H	*BB	*BB	*BB	BBB	BBB	BBB	BBB
0.5	**W	**H	**H	**H	***	***	***	**B	*BB	*BB	*BB	*BB	*BB
0.4	**2	***	***	***	***	***	***	***	**B	**B	*BB	*BB	*BB
0.3	***	***	***	***	***	***	***	***	***	**B	**B	**B	**B
0.2	***	***	***	***	***	***	***	***	***	***	***	***	**B
0.1	***	***	***	***	***	***	***	***	***	***	***	***	***
	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
	360°	345°	330°	315°	300°	285°	270°	255°	240°	225°	210°	195°	180°

desending arc

Figure 15. $q = q(\omega)$, $D \leq 0.2$, inclination $= 35°$or $145°$.

ascending arc

	180°	165°	150°	135°	120°	105°	90°	75°	60°	45°	30°	15°	0°
	180°	195°	210°	225°	240°	255°	270°	285°	300°	315°	330°	345°	360°
1.2	QQQ	***	***	***	***	B**	B**	B**	B**	B**	B**	B**	B**
1.1	QQQ	QQQ	QQQ	BBB	BB*	BB*	B**	B**	B**	B**	B**	B**	B22
1.0	×××	QQQ	QQQ	QQQ	BBB	BBB	BB*	BB*	BB*	WW*	WWW	WWW	×××
0.9	PQQ	QQQ	QQQ	BBQ	BQB	BBB	BBB	BHB	BHH	BBH	EB*	EB2	EB2
0.8	QQQ	QQQ	BQQ	BBB	BBB	BHQ	BBB	BBH	BBB	BBB	BBB	BBB	BBB
0.7	*W*	*W*	*H*	*H*	**B	*BH	BBB	BBB	BBB	BBB	BBB	BBB	BBB
0.6	*2*	*2*	***	**H	**H	**H	*BB	*BB	*BB	BBB	BBB	BBB	BBB
0.5	**W	**H	**H	***	***	***	***	**B	*BB	*BB	*BB	*BB	*BB
0.4	**2	***	***	***	***	***	***	***	**B	**B	*BB	*BB	*BB
0.3	***	***	***	***	***	***	***	***	***	**B	**B	**B	**B
0.2	***	***	***	***	***	***	***	***	***	***	***	***	**B
0.1	***	***	***	***	***	***	***	***	***	***	***	***	***
	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
	360°	345°	330°	315°	300°	285°	270°	255°	240°	225°	210°	195°	180°

desending arc

Figure 16. $q = q(\omega)$, $D \leq 0.2$, inclination $= 50°$or $130°$.

ascending arc

	180°	165°	150°	135°	120°	105°	90°	75°	60°	45°	30°	15°	0°
	180°	195°	210°	225°	240°	255°	270°	285°	300°	315°	330°	345°	360°
1.2	QQQ	***	***	***	***	B**	B**	B**	B**	B**	B**	B**	B**
1.1	QQQ	QQQ	QQQ	BBB	BB*	BB*	B**	B**	B**	B**	B**	B**	B22
1.0	×××	QQQ	QQQ	QQQ	BBB	BBB	BB*	BB*	BB*	WW*	WWW	WWW	×××
0.9	PQQ	QQQ	QQQ	BBQ	BQB	BBB	BBB	BHB	BHH	BBH	EBH	EB*	EB2
0.8	QQQ	QQQ	BQQ	BBB	BBB	BHQ	BBB	BBH	BBB	BBB	BBB	BBB	BBB
0.7	*W*	*H*	*H*	*H*	*HB	*BH	BBB	BBB	BBB	BBB	BBB	BBB	BBB
0.6	*2*	***	***	**H	**H	**H	*BB	*BB	*BB	BBB	BBB	BBB	BBB
0.5	**W	**H	**H	**H	***	***	***	**B	*BB	*BB	*BB	*BB	*BB
0.4	**2	***	***	***	***	***	***	***	**B	**B	*BB	*BB	*BB
0.3	***	***	***	***	***	***	***	***	***	**B	**B	**B	**B
0.2	***	***	***	***	***	***	***	***	***	***	***	***	**B
0.1	***	***	***	***	***	***	***	***	***	***	***	***	***
	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
	360°	345°	330°	315°	300°	285°	270°	255°	240°	225°	210°	195°	180°

desending arc

Figure 17. $q = q(\omega)$, $D \leq 0.2$, inclination = 65°or 115°.

ascending arc

	180°	165°	150°	135°	120°	105°	90°	75°	60°	45°	30°	15°	0°
	180°	195°	210°	225°	240°	255°	270°	285°	300°	315°	330°	345°	360°
1.2	QQQ	***	***	***	***	B**	B**	B**	B**	B**	B**	B**	B**
1.1	QQQ	QQQ	QQQ	BBB	BB*	BB*	B**	B**	B**	B**	B**	B**	B22
1.0	×××	QQQ	QQQ	QQQ	BBB	BBB	BB*	BB*	BB*	WW*	WWW	WWW	×××
0.9	PQQ	QQQ	QQQ	BBQ	BQB	BBB	BBB	BHH	BHH	BHH	EBH	EB*	EB2
0.8	QQQ	QQQ	BQQ	BBB	BHB	BHQ	BBB	BBH	BBH	BBB	BBB	BBB	BBB
0.7	*W*	*H*	*H*	*H*	*HH	*BH	BBB	BBB	BBB	BBB	BBB	BBB	BBB
0.6	*2*	***	**H	**H	**H	**H	*BB	*BB	*BB	BBB	BBB	BBB	BBB
0.5	**W	**H	**H	**H	***	***	***	**B	*BB	*BB	*BB	*BB	*BB
0.4	**2	***	***	***	***	***	***	***	**B	**B	*BB	*BB	*BB
0.3	***	***	***	***	***	***	***	***	***	**B	**B	**B	**B
0.2	***	***	***	***	***	***	***	***	***	***	***	***	**B
0.1	***	***	***	***	***	***	***	***	***	***	***	***	***
	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
	360°	345°	330°	315°	300°	285°	270°	255°	240°	225°	210°	195°	180°

desending arc

Figure 18. $q = q(\omega)$, $D \leq 0.2$, inclination = 80°or 100°.

It was necessary to introduce the distance condition of the whole orbit from the Earth's orbit (0.2 AU) to distinquish whether the orbit geometry or absence of suitable method was the cause for the D-criterion condition not being fulfilled. One can see from the plots that the method of varying both the perihelion distance and eccentricity has the largest application.

4. Comparison of radiants obtained by various methods

A comparison of theoretical radiants obtained by various methods was made for 3 asteroids. Individual asteroids were chosen as typical representatives of each group of the NEA objects (Bacchus as Apollo, Eros as Amor and Aten as Aten). The orbital elements of the asteroids were taken from the Ephemerides of Minor Planets for 1991 (Batrakov et al, 1990).

Table 1. MP 2063 Bacchus - applicability of individual methods

method	*D*	R.A.	Decl.	v
Q	0.105	0.8	+26.9	10.6
E	0.111	352.9	+36.5	7.7
W	0.152	13.6	+33.0	11.2
B	0.078	357.7	+30.9	9.1
A	0.074	351.2	+19.4	10.9
H	0.072	350.8	+21.0	11.0
P	0.081	352.6	+17.9	11.0

Table 2. MP 433 Eros - applicability of individual methods

method	*D*	R.A.	Decl.	v
Q	0.149	327.4	+69.7	6.7
E	-			
W	-			
B	0.149	327.4	+69.7	6.7
A	-			
H	-			
P	0.196	283.5	+69.4	5.9

In the comparison the eccentricity of the Earth orbit was taken into account.

The comparisons of the suitability of individual methods are shown in Tables 1-3. The abbreviations of the methods are the same as used in Section 3,

Table 3. MP 2062 Aten - applicability of individual methods

method	*D*	R.A.	Decl.	v
Q	0.087	133.5	−46.3	10.4
E	0.063	130:0	−53.9	10.0
W	0.131	117.9	−38.2	11.0
B	0.052	131.4	−51.3	10.1
A	0.227	147.7	−31.8	7.9
H	0.206	143.7	−40.2	9.5
P	0.174	140.2	−42.6	10.5

R.A. is the right ascension and *Decl* is declination of a radiant in degrees (equinox 1950.0), v is the geocentric velocity in $km\ s^{-1}$.

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