# Atmospheric extinction at the Brno and Skalnaté Pleso Observatories

## II. Interpretation of observations. Models of extinction. Long-term and seasonal variations. Prediction.

**Z. Mikulášek[1,2], J. Papoušek[3], J. Tremko[4] and J. Žižňovský[4]**

[1] *N. Copernicus Observatory and Planetarium in Brno, Kraví hora 2*
*616 00 Brno, Czech Republic, mikulas@dior.ics.muni.cz*

[2] *Institute of Theoretical Physics and Astrophysics, Masaryk University*
*Kotlářská 2, 611 37 Brno, Czech Republic*

[3] *Mlýnská 56, 602 00 Brno, Czech Republic*

[4] *Astronomical Institute of the Slovak Academy of Sciences*
*059 60 Tatranská Lomnica, The Slovak Republic*
*tremko@ta3.sk, ziga@ta3.sk, respectively*

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**Abstract.** The behaviour of atmospheric extinction at the Skalnaté Pleso Observatory can be satisfactorily described by a standard two-component model consisting of a more or less constant component given namely by Rayleigh scattering and a variable one caused by the scattering of light on dust particles. In order to explain the properties of Brno extinction we introduce an additional, variable "particular component" acting mainly in the *U* and *B* colours. Long-term and seasonal variations of both variable components of extinction in the *B* colour have been thoroughly investigated. The proposed extinction models are confronted with observations in all colours of the *UBV* system. Trends of the *UBVRI* extinctions are discussed.

**Key words:** extinction: atmospheric

## 1. Introduction

This paper (Paper II) deals with the treatment of two extensive collections of *UBV* extinction coefficients obtained as a by-product of observations of variable stars in 1962 – 1995 at the urban Brno Observatory and at the mountain Skalnaté Pleso Observatory. The basic characteristics of both observational stations and the material treated are given in Mikulášek et al. (2000) (hereafter Paper I). The main aim of the present paper is to build an as simple as possible,

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physically reasoned model of atmospheric extinction and the study of seasonal and long-term variations at both observatories.

## 2. Modelling of atmospheric extinction.

### 2.1. The basic premises of modelling extinction

Extinction is a product of the interaction of a cosmic object's light penetrating the Earth's atmosphere. The extinction of light is caused mainly by its scatter and to a lesser extent by its true absorption. Atmospheric extinction itself is contributed to by numerous physical processes, the effectiveness of which generally depend on the wavelength of the light and on the concentration of extinction centres (dust or liquid particles, molecules of air gases, and their clumps). Assuming extinction processes run independently, the resultant extinction coefficients in a particular colour $c$, $K(c)$, representing the extinction of the light, expressed in magnitudes per air mass $X$, is the sum of contributions of the individual extinction processes $K_l(c)$:

$$K(c) = \sum_l K_l(c). \tag{1}$$

Furthermore, from the presumed independence of the acts of scatter and absorption, it follows that the value of extinction caused by an individual process should be linearly dependent on the number of relevant extinction centres in one air mass. For as much as their concentration apparently does not depend on the wavelength of the scattered or absorbed light, it is obvious that contribution to the $l$-th certain extinction process measured in two different spectral regions (colours) $c_i$ and $c_j$ and $K_l(c_j)$ should fulfil a condition:

$$K_l(c_j) \cong \left[\frac{K_l(c_j)}{K_l(c_i)}\right] K_l(c_i), \tag{2}$$

where the constant of proportionality in the square brackets is something like a "material constant" given only by the mechanism of the individual extinction, not by its particular value. The above mentioned supposition does not have to be always fulfilled. Indeed, we could imagine a situation when in a static atmosphere with a time-independent particle concentration a certain progress of their extinction properties takes place. Under such circumstances we should observe not only changes of the extinction in different colours, but some variations of the "material constants", too. Nevertheless, due to the dynamism of the atmosphere in mid-Europe climate conditions, where in the time scale of several days a considerable interchange of the air mass of dissimilar dustiness take place, this more or less hypothetical mechanism should not have any significant influence. Moreover, modelling the atmospheric extinction we shall assume that the above mentioned constant of proportionality referring to the same process, is the same at both the stations studied.

Constants of proportionality in a number of astrophysical cases equals a certain power of the ratio of effective wavelengths of pertinent colours $\lambda_j, \lambda_i$:

$$\left[\frac{K_l(c_j)}{K_l(c_i)}\right] = \left(\frac{\lambda_j}{\lambda_i}\right)^{-\alpha_l}, \tag{3}$$

where $\alpha_l$ is the spectral index of the particular type of extinction.

### 2.2. Rayleigh scattering and absorption by ozone. The constant component of the atmospheric extinction

The scatter on random clumps of molecules of gaseous component of the Earth atmosphere (commonly called as a Rayleigh scattering) is a stable and unavoidable component of the extinction of all Earth-grounded observatories. The Rayleigh scattering is very sensitive to the wavelength: its efficiency is inversely proportional to the fourth power of wavelength of the light going through (Crisp 2000).

The extinction term $R(c)$ represents a more or less constant component of the atmospheric extinction, which is as a first approximation proportional to the number of molecules of air gases contained in an atmospheric column with a base of 1 square metre, consequently to the momental barometric pressure $P(h)$. To the contribution $R(c_i)$ in the $i$-th colour, caused by the Rayleigh scattering, another, not very variable extinction should be added, induced by a true absorbtion in molecular bands of the ozone (the ozone absorbtion in the optical region is significant only in the $V$ colour). The contributions in the $UBV$ colours are commonly given in the form:

$$R(V) = [0.1066P(h)/P(0) + 0.031] \text{ mag}/X \tag{4}$$

$$R(B) = [0.2661P(h)/P(0) + 0.001] \text{ mag}/X \tag{5}$$

$$R(U) = 0.5781P(h)/P(0) \text{ mag}/X. \tag{6}$$

The Rayleigh contribution in a colour, the effective wavelength $\lambda$ of which is more than 550 nm, can be expressed with a sufficient accuracy by the relation:

$$R(\lambda) = 0.107 \left(\frac{\lambda}{550\text{nm}}\right)^{-4} \frac{P(h)}{P(0)} \text{ mag}/X. \tag{7}$$

Particularly, for the colours $R$ and $I$ of the standard broadband $UBVRI$ photometric system we can write:

$$R(R) = 0.041\ P(h)/P(0) \text{ mag}/X \tag{8}$$

$$R(I) = 0.016\ P(h)/P(0) \text{ mag}/X \tag{9}$$

The mean ratio of the barometric pressure $P(h)$ on the station of the elevation $h$ (in meters) above sea level to the pressure at sea level $P(0)$ can be approximated using a simple model of the so called standard Earth atmosphere with a scale height of $H = 7996$ m.

$$P(h)/P(0) = \exp\,(-h/7996) \tag{10}$$

The ratio is thus a function of the altitude $h$ of the observing station. The altitude of the Brno Observatory is: $h(\mathrm{B}) = 310$ m above sea level, and the altitude of the Skalnaté Pleso Observatory is: $h(\mathrm{P}) = 1783$ m above sea level. The reliability of the relation (10) was tested in the case of the Skalnaté Pleso Observatory, where regular measurements of the air pressure has been performed for many years. Measurements done in the last five years (Jakubjak 1997) establishes that our approximation of the situation by a standard atmosphere model are appropriate for our purposes. Expected mean values of the Rayleigh extinction coefficients are given in Table 1.

**Table 1.** Comparison of the constant Rayleigh component in the *UBV* system with the variable component of extinction.

| Color (effective wavelength) | $V$ (550 nm) | | $B$ (440 nm) | | $U$ (365 nm) | |
|---|---|---|---|---|---|---|
| Station | Brno | Sk. Pl. | Brno | Sk. Pl. | Brno | Sk. Pl. |
| Number of measurements | 379 | 366 | 344 | 248 | 290 | 152 |
| $R(c)$ in mag/$X$ | 0.134 | 0.116 | 0.257 | 0.213 | 0.556 | 0.463 |
| Average $K'$ in mag/$X$ | 0.311 | 0.130 | 0.446 | 0.165 | 0.559 | 0.218 |
| $SD$ in mag/$X$ | 0.17 | 0.19 | 0.21 | 0.23 | 0.24 | 0.26 |

The actual value of $R(c_i)$ depends on the instant value of the barometric pressure, or on the momentary thickness of the ozone layer. However, as relative variations of the barometric pressure in the course of the year are insignificant ($< 4\%$), and the absorbtion in the ozone layer is negligible in the optical region, we can consider the $R(c_i)$ component of the extinction to be effectively constant. Omitting the variability in the air pressure and in the thickness of the ozone layer we introduce a noise, the amplitude of which is by one or more orders smaller than the noise caused by the uncertainty of extinction measurements.

Assuming $R(c)$ to be constant, it will be advantageous to correct all the observed extinction coefficients $K(c)$ by it and to pass to the so called corrected extinction coefficients $K'(c)$:

$$K'(c) \;=\; K(c) - R(c) \tag{11}$$

The cursory comparison of data given in Tables 1 and 2 of Paper I leads to the following conclusions: 1) the mean values of corrected extinction coefficients are comparable to Rayleigh's ones, 2) corrected extinction coefficients vary to a considerable extent. It is evident that besides the constant component of the extinction, there are very prominent, more or less variable sources of atmospheric extinction.

### 2.3. Relationships among the extinction coefficients. The two-component model of extinction

Basic information on the nature of the variable component (or variable components) of the atmospheric extinction in Brno and Skalnaté Pleso offers mutual relationships among extinction coefficients obtained simultaneously in different colours of the *UBV* photometric system. As we have shown in Paper I (see Fig. 4 therein): 1) correlations between the individual extinction coefficients are firm and evidently linear, 2) the fitting lines for Brno and Skalnaté Pleso are parallel within the frame of observational errors.

We shall attempt to interpret the above described behaviour of extinction by a simple linear two-component model of atmospheric extinction, which consists of a relatively constant component $R(c_i)$, determined namely by the Rayleigh scattering and ozone absorbtion, and a variable component $D(c_i)$ caused by scatter on particles of various origin in the Earth atmosphere. Because there are no prominent absorbtion bands of water vapour in all of the *UBV* colours, we will not consider the influence of the otherwise strongly variable intensity of water absorbtion.

The extinction coefficient in colour $c$, $K(c)$ is the sum of contributions of both the above-mentioned components:

$$K(c) \cong R(c) + D(c) \quad \Rightarrow \quad K'(c) = K(c) - R(c) \cong D(c) \tag{12}$$

Assuming the coherency of dust particles in the process of atmospheric extinction described by the relation (2) we arrive at the expression:

$$K'(c_j) \cong \left[\frac{D(c_j)}{D(c_i)}\right] D(c_i) \;=\; \left[\frac{D(c_j)}{D(c_i)}\right] K'(c_i) \tag{13}$$

Using the proposed simple two-component extinction model, the graph of the relationship among the corrected extinction coefficients $K'(c_j)$ vs. $K'(c_i)$ should be a straight line passing through the origin. The slope of the $(a_{ji})$ can be expressed (in accordance with the relation (3) by means of the parameter $\alpha_{ji}$:

$$\alpha_{ji} \;=\; \frac{\mathrm{d}K'(c_j)}{\mathrm{d}K'(c_i)} = \left[\frac{D(c_j)}{D(c_i)}\right] = \left(\frac{\lambda(c_j)}{\lambda(c_i)}\right)^{-\alpha_{ji}} \quad \Rightarrow \quad \alpha_{ji} = \frac{\log \alpha_{ji}}{\log\left[\lambda(c_i)/\lambda(c_j)\right]} \tag{14}$$

The slopes of relationships for all the three combinations of colours of the *UBV* system have been found by means of orthogonal regression supposing the relevant slopes for both observational stations are identical. Values of the slope $(a_{ji})$ and corresponding spectral parameters $\alpha_{ji}$ are given in Table 2. It seems that all the three values found for $\alpha$ are effectively identical and we are justified in putting the parameter $\alpha$ to be equal to its mean value $\alpha = 1.04 \pm 0.04$ and then: $[D(V)/D(B)] = 0.793 \pm 0.007$, $[D(U)/D(V)] = 1.532 \pm 0.026$, and $[D(U)/D(B)] = 1.214 \pm 0.009$. The noticeable proximity of the parameter $\alpha$ to 1.0 indicate that the prevailing part of the time-variable extinction is caused by the classic Mie scattering on dielectric dust particles (aerosols) of dimensions comparable to the wavelength of optical light (Gutierrez-Moreno et al. 1982). For that reason, we shall henceforth call the $D(c)$ component of extinction the *dust extinction*.

As the spectral properties of the variable extinction at the Brno and Skalnaté Pleso Observatories are almost identical, we can anticipate that the cause of it

**Table 2.** Determination of the $\alpha$ parameter from the slopes of relations among extinction coefficients obtained in various colours simultaneously.(For both observatories.)

| | $\lambda(c_i)/\lambda(c_j)$ | No. of pairs | $r$ | $a_{ij}$ | $\alpha$ |
|---|---|---|---|---|---|
| $K'(B)$ vs. $K'(V)$ | 550/540 | 498 | 0.925 | 1.224±0.020 | 0.98±0.07 |
| $K'(U)$ vs. $K'(V)$ | 550/365 | 429 | 0.944 | 1.532±0.032 | 1.04±0.05 |
| $K'(U)$ vs. $K'(B)$ | 440/365 | 430 | 0.956 | 1.224±0.018 | 1.08±0.08 |

might be the same. We try to identify if it may be by the scattering on tiny dust particles dragged there by winds or by a smoke from sources related to human activity. We believe that the disparity of the astroclimate of the stations is not the result of different quality of the extinction processes, but the result of their dissimilar quantity is caused by their different altitudes.

Values of the parameter $\alpha$ presented in literature differ considerably: Angstroem (1961) denotes that $\alpha = 1.3$, while Tueg et al. (1977) and Hayes & Lantham (1975) conversely judge that the typical value of the parameter $\alpha$ is 0.9.

### 2.4. The three-component model of extinction

If we fit the observed relationships between the corrected extinction coefficients for the Skalnaté Pleso Observatory by lines with slopes determined by the spectral coefficient $\alpha = 1.04 \pm 0.04$ found for the dust component of extinction, we obtain:

$$K'(B) = (1.261 \pm 0.011)[K'(V) - 0.132] + (0.168 \pm 0.009)$$
$$\Rightarrow \text{for} \quad K'(V) = 0 \quad K'(B,0) = (0.002 \pm 0.009)\ \text{mag}/X$$
$$K'(U) = (1.532 \pm 0.026)[K'(V) - 0.132] + (0.212 \pm 0.015)$$
$$\Rightarrow \text{for} \quad K'(V) = 0 \quad K'(U,0) = (0.004 \pm 0.016)\ \text{mag}/X$$
$$K'(U) = (1.214 \pm 0.009)[K'(B) - 0.168] + (0.212 \pm 0.015)$$
$$\Rightarrow \text{for} \quad K'(B) = 0 \quad K'(U,0) = (0.008 \pm 0.015)\ \text{mag}/X$$

For the purpose of fitting, only measurements taken in nights when extinction was measured in all three colours of the *UBV* system (142 threes) were used. It is evident that the fitting lines pass very near to the origin of the corrected coordinates which means that the relations (13) are valid and the two-component model of extinction can be considered as applying to the case of the Skalnaté Pleso Observatory.

Rendering the observed scattering around the fitting lines primarily as the cause of observational errors, we can conclude that the standard error of determination of one extinction coefficient at the Skalnaté Pleso Observatory represents 0.10 mag/$X$. We must remind the reader of the fact that this error refers to the cases when extinction was measured in all three colours. However, measurements as follows, have been carried out since 1975, when purposive extinction measurements started. Detailed treatment of the material displays that measurements of extinction obtained earlier were loaded by roughly a twofold error.

This is true for more than 45 % of measurements and we estimate the standard error of one measurement of Skalnaté Pleso extinction reaches 0.15 mag/$X$.

Fitting now the observed relationships among the corrected extinction coefficients measured in individual colours at the Brno Observatory by lines of the same slopes, we obtain:

$$K'(B) = (1.261 \pm 0.011)[K'(V) - 0.296] + (0.453 \pm 0.004)$$
$$\Rightarrow \text{for} \quad K'(V) = 0 \quad K'(B,0) = (0.080 \pm 0.005)\ \text{mag}/X$$
$$K'(U) = (1.532 \pm 0.026)[K'(V) - 0.296] + (0.553 \pm 0.006)$$
$$\Rightarrow \text{for} \quad K'(V) = 0 \quad K'(U,0) = (0.100 \pm 0.010)\ \text{mag}/X$$
$$K'(U) = (1.214 \pm 0.009)[K'(B) - 0.453] + (0.553 \pm 0.006)$$
$$\Rightarrow \text{for} \quad K'(B) = 0 \quad K'(U,0) = (0.003 \pm 0.006)\ \text{mag}/X$$

We have again used only such measurements, when during the individual night extinction measurements were performed in all three colours (285 threes). Interpreting the observed scatter around the fitted lines as uncertainties of observation, we can estimate that the standard error of the determination of extinction coefficient in Brno represents $\approx 0.05$ mag/$X$.

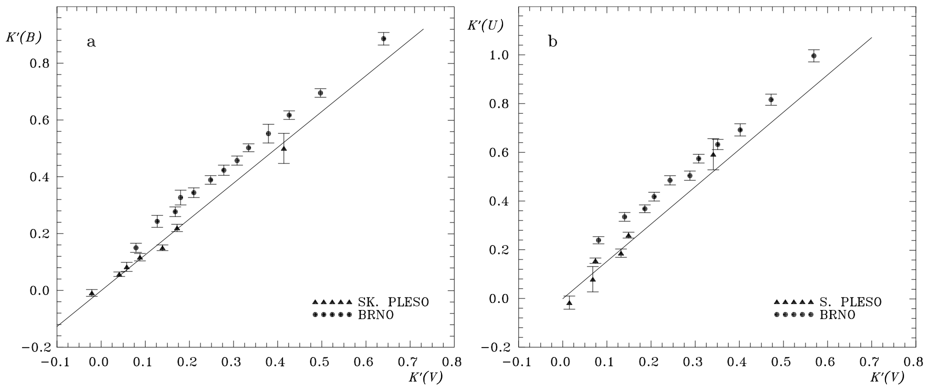



**Figure 1.** Relation between the corrected extinction coefficients $K'(B)$ and $K'(V)$ **(a)**, $K'(U)$ and $K'(V)$ **(b)**, represented by normal points (each of them corresponds to $\approx 25$ measurements). The line passing through the origin of coordinates represents the two-component model with dust particles, whose extinction is described by the spectral parameter $\alpha = 1.04$. Spacing of the Brno measurements from this model line is evident in both cases.

It is evident (see Fig. 1) that the fitted lines generally do not pass through the origin of coordinates. The difference is larger than $10\,\sigma$ ! It is obvious that in the case of Brno atmosphere a simple linear two-component model is inadequate. The team of Papoušek et al. (1984) arrived at the same conclusion studying the properties of extinction in Brno atmosphere (stemming, of course, from a smaller amount of observational material).

Several explanations for the discrepancy can be used. At first, we shall introduce one, trying to hold the two-componentness of the extinction model: atmospheric extinction is the result of scattering on molecules of air gases and on aerosols.

1) If we want to save the linearity of mutual relationships among the extinction coefficients as it follows from the observations, we have to permit that our assumption of the value of the constant – Rayleigh component $R(c)$ in Brno might not be correct. This component would have to be at least by $(18 \pm 2)\%$ in the $V$ and even by $(31 \pm 2)\%$ in the $U$ and $B$ colour, respectively, greater than relations (4) – (6) denote. Let us to remark, that these relations has not been challenged by anybody up to now, moreover they fit observations done at the Skalnaté Pleso Observatory and elswhere very well.

2) Conversely, if we suppose that our estimate of the value of Rayleigh component of extinction $R(c_i)$ is proper, then the equation: $D(c) = K(c)$ is valid. In such a case, however, the assumption of mutual proportionality among the corrected extinction coefficients in the individual colours of the $UBV$ system can not be fulfilled and then neither the ratio $D(c_j)/D(c_i)$ nor the corresponding parameter $\alpha'_{ji}$ can be constant. The value of $\alpha'_{ji}$ can be evaluated according to the ralation:

$$\alpha'_{ji} \;=\; \log\left[\frac{(D(c_j)}{D(c_i)}\right] \Big/ \log\left(\frac{\lambda_i}{\lambda_j}\right) = \log\left[\frac{(K'(c_j)}{K'(c_i)}\right] \Big/ \log\left(\frac{\lambda_i}{\lambda_j}\right) \qquad (15)$$

Here we arrive at a contradiction: as far as we determine the value of the spectral parameter $\alpha$ by means of the slopes of relationships: $K'(B)$ vs. $K'(V)$ and $K'(U)$ vs. $K'(V)$, then we arrive at the conclusion that $\alpha$ is constant in the whole range of the observed extinction and the same for both relationships. The value of $\alpha = 1.04 \pm 0.04$ corresponds well to the value of the expected Mie scattering. If we use the ralation (18) for evaluation of the same parameter, we see a quite different picture: whilst the parameter $\alpha'_{UB}$ does not change significantly, and its value remains around 1.04, the parameters $\alpha'_{BV}$ and $\alpha'_{UV}$ monotically decrease within the region well documented by the observations (see Fig. 2). Moreover: for the same value of $K'(V)$ an inequality $\alpha'_{BV} > \alpha'_{UV} > \alpha'_{UB}$ was found. It would be very difficult to find any physically evincible extinction mechanism explaining such a complex behaviour of the spectral parameter $\alpha$.

Nevertheless, there is another, much simpler explanation of the nature of the atmospheric extinction at the urban observatory in Brno: to supply the proposed linear model of extinction by a next, independent component which can be either constant or variable.

Let we assume that the atmospheric extinction is a result of superposition of three mutually independent components:

$$K(c) \cong R(c) + D(c) + E(c),$$

where the meaning of homonymous terms is the same as in the two-component model. The third component $E(c)$ is a quite new component of extinction not referred to in previous publications, which we shall call *particular extinction.*

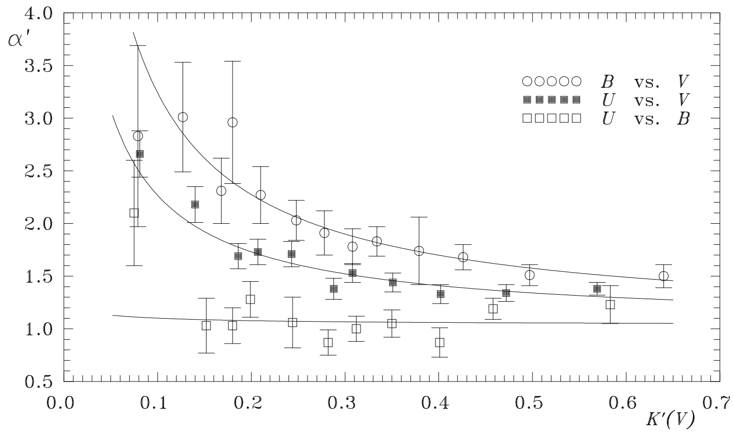



**Figure 2.** Relationships between the fictive spectral parameter $\alpha'$ (computed accordig to relation (18) for the Brno two-component model) and the corrected extinction coefficient $K'(V)$. Normal points were used as in Fig. 1. The behaviour of the $\alpha'$ parameter is based on the assumption that the real relationships between the corrected extinction coefficients are straight lines with slopes of the value $\alpha = 1.04$, which do not pass through the origin of the coordinates.

At first we supposed that the particular extinction represents a more or less constant contribution to the total extinction, which is symptomatic for the atmosphere of an observational station within the city (see Mikulášek et al., 1995). From the physical point of view, it is very difficult to guess a "constant" extinction mechanism sustainable active in Brno and inactive at the Skalnaté Pleso. The assumption that the term $E(c)$ is a quantity, the value of which is time-variable, seems to be much more congenial. Its value would then also be proportional to the number of commensurate hypothetical particles in one air mass and the number of these particles could be variable in diverse time scales. We will support this hypothesis in the following text by several consequential arguments.

Hereafter we shall suggest that the condition (2) is valid for particular extinction, too:

$$E(c_j) \cong \left[\frac{E(c_j)}{E(c_i)}\right] E(c_i), \tag{16}$$

where the expression in squared brackets is a "material constant" independent on the actual value of $E(c)$. The resultant extinction coefficient is the sum of contributions of all three proposed sources of extinction active in the colour $c$:

$$K(c_j) \cong R(c_j) + \left[\frac{D(c_j)}{D(c_i)}\right] D(c_i) + \left[\frac{E(c_j)}{E(c_i)}\right] E(c_i) \tag{17}$$

$$\Rightarrow K'(c_j) \cong \left[\frac{D(c_j)}{D(c_i)}\right] D(c_i) + \left[\frac{E(c_j)}{E(c_i)}\right] E(c_i) \tag{18}$$

The situation e.g. on the plane of diagram $K'(B)$ vs. $K'(V)$ (see Fig. 3a) can be demonstrated to advantage by formally introduced vectors of the dust extinction $\boldsymbol{D}$, with components $[D(V), D(B)]$ and a particular extinction $\boldsymbol{E}$, with components $[E(V), E(B)]$. The observed corrected extinction is in the plane of diagram represented by a vector $\boldsymbol{K'}$ with components $[K'(V), K'(B)]$, consequently: $\boldsymbol{K'} = \boldsymbol{D} + \boldsymbol{E}$. If the condition of coherency of the extinction processes expressed by the relation (2) is valid, then the directions of both vectors $\boldsymbol{D}$ and $\boldsymbol{E}$ are established. When we know them, we can split the vector $\boldsymbol{K}$ into them and thus to determine both vectors $\boldsymbol{D}$ and $\boldsymbol{E}$. If the particular extinction did not exist, then all vectors $\boldsymbol{K'}$ would be co-linear and their ending points would form a ray passing through the origin of the coordinate system. The slope of the semiline is determined by the known spectral parameter $\alpha$. If the vector of the particular extinction $\boldsymbol{E}$ is nonzero, then the ending points of vectors $\boldsymbol{K'}$ will lay generally above this line (see Fig. 3b).[1]

The vertical component of the distance of the observed point to the dust line $\Delta K'(B)$ or $\Delta K'(U)$ can be measured. We can write according to the relation (18):

$$\Delta K'(B) = K'(B) - \left[\frac{D(B)}{D(V)}\right] K'(V) = \left\{ \left[\frac{E(B)}{E(V)}\right] - \left[\frac{D(B)}{D(V)}\right] \right\} E(V), \qquad (19)$$

$$\Delta K'(U) = K'(U) - \left[\frac{D(U)}{D(V)}\right] K'(V) = \left\{ \left[\frac{E(U)}{E(V)}\right] - \left[\frac{D(U)}{D(V)}\right] \right\} E(V). \qquad (20)$$

Comparing relations (23) and (24) we see that quantities $\Delta K'(B)$ and $\Delta K'(U)$ should be mutually proportional. If the suggested extinction model is appropriate, we should find on the graph of relationship of mentioned quantities either 1) a more of less circular cloud of points if the particular extinction is nearly constant, or 2) a prolate cloud of points gathered along the line passing through the origin. Inspecting Fig. 4 we can conclude that the second item is true: the graph of relationship is apparently a line passing through the origin, the correlation coefficient of which is 0.86! This fact can be considered as an important argument in our suspicion that besides the variable dust extinction, there is another unattached variable component of extinction in the Brno atmosphere.

Further analysis displays that the mean particular extinction $E(c)$ in all measured colours is considerably smaller than the corresponding average dust $D(c)$ extinction. On account of it, the presence of particular extinction will manifests itself as a certain shift of relationships among the corrected extinction coefficients $K'(c_j)$ vs $K'(c_i)$, namely by a vector $\boldsymbol{E}$, the components of which are the mean values of the particular extinction in the relevant colours

[1] A special case represents the situation when the slopes of vectors of the dust and particular extinction are by chance either identical or very similar. In such a case the distinguishing of individual extinctions is virtually unrealizable. It occurs in the case of the diagram $K'(U)$ vs. $K'(B)$.

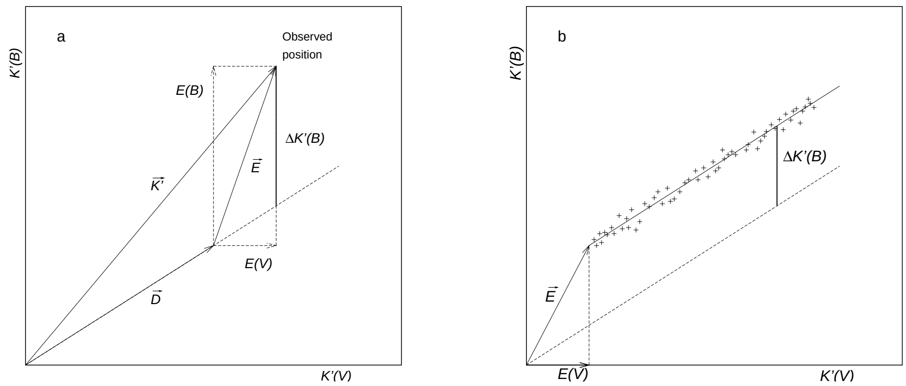



**Figure 3. a)** Schematic view of the situation in the plane of the diagram of the corrected extinction coefficients $K'(B)$ vs. $K'(V)$ in the three-component model. $\boldsymbol{D}$ is the vector of the dust extinction component, $\boldsymbol{E}$ the vector of the particular component and their sum $\boldsymbol{K'}$, whose final point is the observed combination $K'(B)$ vs. $K'(V)$. $\boldsymbol{E(V)}$ and $\boldsymbol{E(B)}$ are the components of the vector $\boldsymbol{E}$, $\Delta K(B)$ is the measurable deflection from the direction of the vector $\boldsymbol{K'}$. **b)** Real situation with different combinations of the dust and particular components. As the value of the dust component is usually greater, the relationships is represented by a ray parallel with the vector $\boldsymbol{K'}$, shifted by the vector $\boldsymbol{E}$ of the mean value of the particular extinction. Its direction can be estimated by inspection of the distribution of observed points in the vicinity of the origin of diagram.

(see Fig. 3b). After elimination of the variable component $D(V)$ we can infer the form of the proposed mean relationships among the observed extiction coefficients and compare it with the reality:

$$K'(B) = \left[\frac{D(B)}{D(V)}\right] K'(V) + \overline{\Delta K'(B)}, \tag{21}$$

$$\overline{\Delta K'(B)} \ = \ \left\{\left[\frac{E(B)}{E(V)}\right] - \left[\frac{D(B)}{D(V)}\right]\right\} \overline{E(V)} = (0.080 \pm 0.005)\mathrm{mag}/X \tag{22}$$

$$K'(U) = \left[\frac{D(U)}{D(V)}\right] K'(V) + \overline{\Delta K'(U)}, \tag{23}$$

$$\overline{\Delta K'(U)} = \left\{\left[\frac{E(U)}{E(V)}\right] - \left[\frac{D(U)}{D(V)}\right]\right\} \overline{E(V)} = (0.100 \pm 0.010)\mathrm{mag}/X \tag{24}$$

There are two, mutually independent equations (22) and (24), but three quantities are unknown. The third bounding condition might be a suggestion

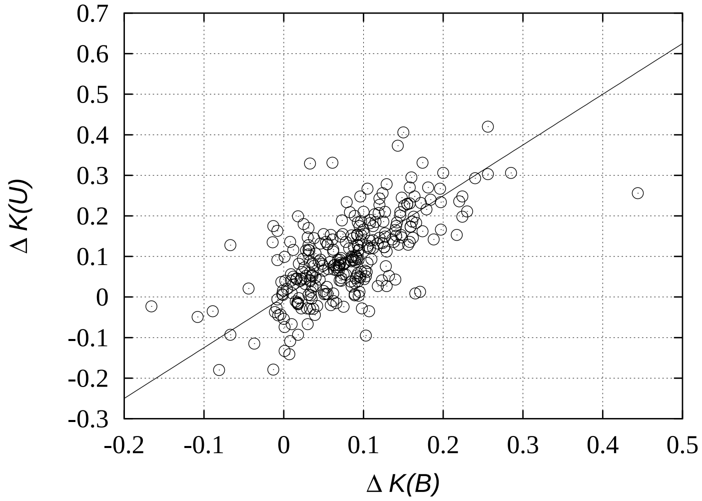



**Figure 4.** The dependence of the values of $\Delta K'(U)$ on $\Delta K'(B)$. Despite of the relatively large scatter, it is evident that there is a clear correlation, what can be interpreted by the variability of the particular extinction.

that even for the particular extinction the exponential law of type (3) is valid similarly as in the case of the other two extinction components:

$$\left[\frac{E(c)}{E(V)}\right] = \left(\frac{\lambda(c)}{\lambda(V)}\right)^{-\beta}. \tag{25}$$

Unfortunately, the system of equations (22), (24) and (25) has no non-trivial solution, which would simultaneously match up to condition that the mean value of the particular extinction in $V$ must not be negative: $\overline{E(V)} \geq 0$. We can therefore conclude that the particular extinction does not obey the exponential law of relation (25) type. At the same time, the negative result of our attempt appears to suggest that the particular extinction is not caused by classical scattering, but more likely by a selective true absorbtion, maybe in some unspecified molecular bands.

So we can either 1) resign ourselves to not determining explicitly the values of particular components of extinction in individual colours and instead of them to investigate the behaviour of the directly measurable quantities $\Delta K'(B)$ and $\Delta K'(U)$ (see relations (19) and (20)), which are pertinently proportional to the particular extinction $E(V), E(B)$ or $E(U)$, 2) attempt to estimate the value of ratios $[E(c)/E(V)]$ bounded by the relation (25), or the mean value of the particular extinction in one colour, e.g. $\overline{E(V)}$.

For a more vivid description of the situation, we have come to the second approach, recognizing however the first approach to be more rigorous. Our estimate of ratios $[E(c)/E(V)]$ insists on a parallel analysis of the dependence of the observed ratios $K(B)/K(V)$ and $K(U)/K(V)$ on $K'(V)$ for the part of observations with smallest values of extinction coefficients, where (according to our model) the contribution of the dust extinction can be neglected (Fig. 3b). We have arrived at the following, more or less informative estimation of the mean value of the particular extinction in the $V$ colour, $\overline{E(V)}$:

$$\overline{E(V)} = (0.030 \pm 0.015)\mathrm{mag}/X \tag{26}$$

By means of relations (26) and (28) we can determine both the mean value of the particular extinction in the remained two colours and the ratios of the particular extinctions in different colours:

$$\overline{E(B)} = (0.118 \pm 0.020)\mathrm{mag}/X, \overline{E(U)} = (0.146 \pm 0.025)\mathrm{mag}/X, \tag{27}$$

$$\left[\frac{E(V)}{E(B)}\right] = (0.26 \pm 0.09), \left[\frac{E(U)}{E(V)}\right] = (4.9 \pm 1.7), \left[\frac{E(U)}{E(B)}\right] = (1.24 \pm 0.07). \tag{28}$$

As follows from the above relations, the relative uncertainty of the determination of the mean value of the particular extinction in $B$ and $U$ is apparently smaller than in $V$. This lead us to a decision to study variations of the variable components of extinction in colour $B$. Knowing now the parameters encountered in the basic equation of the three-component extinction model (18), we can express the coefficients of the particular extinction $E(B)$ and of the dust extinction $D(B)$ in the $B$ colour by a linear combination of the corrected extinction taken in a pair of colours, $c_1$ and $c_2$:

$$E(B, c_1, c_2) = G(c_1, c_2)\left\{\left[\frac{D(c_1)}{D(B)}\right] K'(c_2) - \left[\frac{D(c_2)}{D(B)}\right] K'(c_1)\right\}, \tag{29}$$

$$D(B, c_1, c_2) = G(c_1, c_2)\left\{\left[\frac{E(c_2)}{E(B)}\right] K'(c_1) - \left[\frac{E(c_1)}{E(B)}\right] K'(c_2)\right\}, \tag{30}$$

$$G(c_1, c_2) = \left\{\left[\frac{D(c_1)}{D(B)}\right]\left[\frac{E(c_2)}{E(B)}\right] - \left[\frac{D(c_2)}{D(B)}\right]\left[\frac{E(c_1)}{E(B)}\right]\right\}^{-1}. \tag{31}$$

After substitution of values of the parameters, we arrive at the following values of the quantity $G$: $G(VB) = 1.85 \pm 0.30, G(VU) = 1.50 \pm 0.27, G(BU) = 110 \pm 170$. Since the relative uncertainty of $G(BU)$ is too high, we have not used the pairs $K(U)$, $K(B)$ for the determination of the dust and particular extinction variability.[2]

[2]The enormous uncertainty of the parameter $G(BU)$ is a consequence of the fact that the ratios $[E(U)/E(B)]$ and $[D(U)/D(B)]$ are very close one to another, which leads to the indistinguishability of the dust and particular extinctions.

## 3. Variations of the variable components of extinction

### 3.1. Description of the variations

The studied seasonal and long-term variations of the mean value of the variable components of extinction have been considered as mutually independent modulations of a certain mean value quantified by its arithmetic average $a_0$:

$$M(a_0, \{b_i\}, t, \{B_j\}, T) = a_0[1 + \sum_i b_i f_i(t)][1 + \sum_j B_j F_j(T)], \tag{32}$$

The expression in the first of the squared brackets represents the relative long-term evolution commensurate with the time $t$, expressed in years and their fractions. The expression in the second of brackets describes the relative seasonal variations, where $T = \mathrm{Frac}(t)$. If relative variations are comparably small in respect to 1, the above mentioned model will transform into fully linearized form:

$$M(a_0, \{b_i\}, t, \{B_j\}, T) = a_0[1 + \sum_i b_i f_i(t) + \sum_j B_j F_j(T)], \tag{33}$$

coefficients of which can be easily found by a standard LSM linear regression. In the opposite case, we have to proceed by the method of subsequent, usually quickly convergent iterations.

The sets of functions used, $\{1, f_i(t)\}$ and $\{1, F_i(T)\}$ were constructed so that they mutually fulfil the condition of orthogonality on the investigated ensemble of $N$ measurements, hence for $i \neq j$:

$$\sum_{k=1}^{N} f_i(t_k) = \sum_{k=1}^{N} f_i(t_k) f_j(t_k) = \sum_{k=1}^{N} F_i(T_k) = \sum_{k=1}^{N} F_i(T_k) F_j(T_k) = 0. \tag{34}$$

We shall occupationally call such a type of the regression model as "a quasiorthogonal model". The quasiorthogonal set of all time-dependent functions $\{1, f_i(t), F_j(T)\}$ used in the further treatment are nearly orthogonal as a whole[3], which is a consequence of the fact that the distribution of clear nights during the year does not exhibit any long-term trend.

The benefit of the usage of orthogonal set of functions consists of the fact that the decomposition of the observed dependence into individual components in the space of functions is mutually independent. It enables us to discuss the true errors of the found parameters of regression models and to evaluate the errors of the forecast straightforwardly. What is disadvatageous in this approach, is a failure of lucidity when there are too many free parameters in our linear regression model.

For the description of the course of long-term variations we prefered to use such functions $f_i(t)$, which have their inflection both in the beginning and in the end of the described interval. This property enables a sober forecast of the behaviour of the studied dependency in the regions not covered by observations.

[3]It denotes mathematically: $\mid \sum_{k=1}^{N} f_i(t_k) F_j(T_k) \mid << \sqrt{\left[\sum_{k=1}^{N} f_i^2(t_k)\right] \left[\sum_{k=1}^{N} F_i^2(T_k)\right]}$

### 3.2. Variations of the particular extinction – Brno

Coefficients of the particular extinction component in the $B$ colour have been evaluated by means of the following relations, which are the straight consequence of general relations (29) and (31):

$$E(B, VB) = 1.47\Delta K'(B) = 1.47K'(B) - 1.85K'(V) \tag{35}$$

$$E(B, VU) = 1.18\Delta K'(U) = 1.18K'(U) - 1.81K'(V) \tag{36}$$

As there are 611 pairs of extinction coefficients measured in both colours $V$ and $B$ or $V$ and $U$, we have obtained 611 coefficients of particular extinction. The characteristics of the material are presented in Table 3. The distribution function (Fig. 5a) is only mildly asymmetric ($z = 0.05$) and do not exhibit seasonal or secular variations. The participation of the particular extinction on the whole variable component of extinction is slight in the $V$ – 10%, it is much pronounced in both the $B$ and $U$ colours, where it reaches identically 27%.

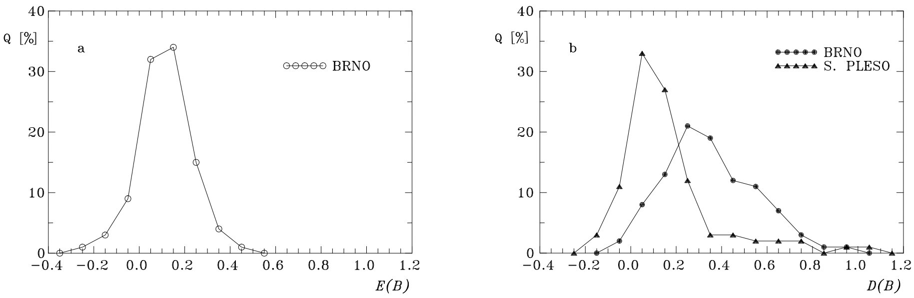



**Figure 5.** The distribution functions of the variable components of extinction: **a)** the particular extinction $E(B)$ and **b)** the dust extinction. $Q$ is the rate (in %) of the extinctions in the interval 0.1 mag/$X$ wide.

The match of the nature of variations of this up now unknown type of extinction could represent an important moment in our attempt to understand its nature. It seems that the particular extinction exhibits both long-term and seasonal variations. The observed time-dependence of the mean value of the particular extinction can be expressed by a quasiorthogonal model in the form:

$$E(B, t) = a_0\{1 + B_1[\cos(2\pi(t - 0.0355)) + 0.380] + B_2[\sin(2\pi(t - 0.0355)) + 0.054]\}$$

$$\{1 + b_1[\tau - 0.684] + b_2[\sin(\pi\tau) + 0.682\tau - 1.061]\}, \tag{37}$$

where $\tau = (t - 1964.44)/30.14$. The following parameters, including their errors, have been found by an iterative least square method:[4] $a_0 = (0.112 \pm 0.005)$mag/$X$, $B_1 = -0.26 \pm 0.08$, $B_2 = -0.01 \pm 0.06$, $b_1 = 0.09 \pm 0.17$, $b_2 = 0.87 \pm 0.16$. Mainly long-term variations participate in the proved variations of the particular extinction ($\approx 23\%$!). There are undoubtedly seasonal variations, too: the square root of the mean value of squares of these variations is 13%.

[4]Four points exhibiting very large deviations from the observed time dependencies were eliminated from further treatment.

**Table 3.** Characterization of the variable extinction in the $B$ colour.

| Station | Brno | | Sk. Pl. |
|---|---|---|---|
| Type of extinction | particular | dust | |
| Median year | 1986.6 | 1985.7 | 1978.0 |
| Average year | 1985.0 | 1983.5 | 1977.6 |
| Num. of measurements | 611 (607) | 685 | 766 |
| Ext. components (mag/$X$) | $E(B)$ | $D(B)$ | $D(B)$ |
| Average extinction | 0.115 | 0.352 | 0.168 |
| $E(B, H = 1/6), D(B, H = 1/6)$ | 0.012 | 0.149 | 0.017 |
| $E(B, H = 1/2), D(B, H = 1/2)$ | 0.110 | 0.322 | 0.109 |
| $E(B, H = 5/6), D(B, H = 5/6)$ | 0.218 | 0.561 | 0.274 |
| Width of distribution function $W$ | 0.206 | 0.412 | 0.257 |
| Asymmetry of distribution function $z$ | 0.05 | 0.16 | 0.28 |

The long-term course of the particular extinction $E(B, t)$ corrected by the influence of seasonal variations is apparent immediately (see Fig. 6a): from the very beginning we observe a certain increase to a flat maximum of $(0.139 \pm 0.008)$ mag/$X$ in $1981.9 \pm 0.6$, with a decrease to $(0.077 \pm 0.011)$ mag/$X$ at the end of the pursued interval (1994). We argue the cause of the particular extinction obviously originated in the mid-sixties, it culminated in the eighties and it is declining now.

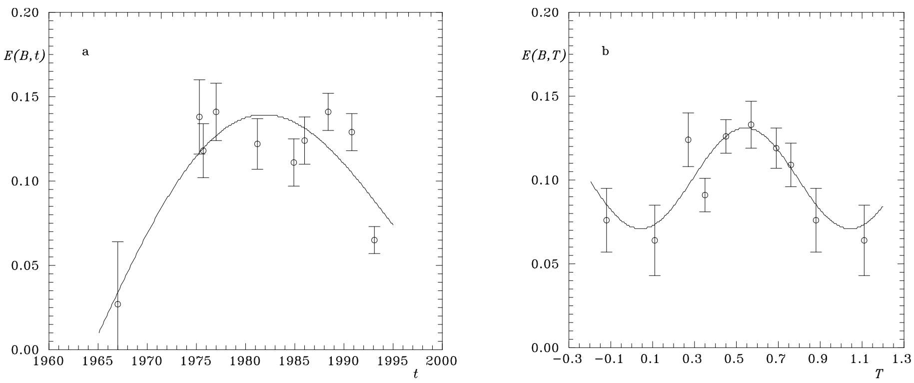



**Figure 6. a)** Long-term variability of the mean value of the particular extinction in the $B$ colour, corrected for seasonal changes. **b)** Seasonal variations of the particular extinction in the $B$ colour, corrected for long-term trends, expressed in mag/$X$, $T$ – fractions of the year. Normal points, each representing a mean value of more than 60 individual measurements, were used. The fitting cuves were computed according to relation (37).

During the year we observe a more or less sinusoidal variation of the particular extinction (see Fig. 6b) with a maximum $(0.131 \pm 0.008)$ mag/$X$ reached around $(16 \pm 13)$ July. The datum corresponds with the maximum of the average day temperature, which takes place in Central Europe with a delay of $\approx 30$ days after the maximum insolation. The minimum value $(0.072 \pm 0.013)$ mag/$X$ occurs a half a year later. The seasonal curve duplicates the seasonal variations of the temperature of the local ground layer of the atmosphere. Nevertheless, it is possible that the particular extinction does not depend on the temperature instantly, but vicariously, e.g. on the car traffic density. It is meritorious to remark that the confidence of either seasonal or a long-term variability, which display a completely different course from the course of the dust component, poses a strong argument for the existence of this component.

### 3.3. Variation of the dust extinction – Brno

Values of the dust component of the Brno extinction in the $B$ colour have been calculated applying relations (30), (31) and (37) by means of which the contribution of the particular extinction has been eliminated:

$$D(B, VB) = 1.85K'(V) - 0.47K'(B), \quad D(B, VU) = 1.86K'(V) - 0.39K'(U),$$

$$D(B, BU) = 0.5(D(B, B) + D(B, U)) = 0.50K'(B) + 0.41K'(U) - 1.01E(B, t),$$

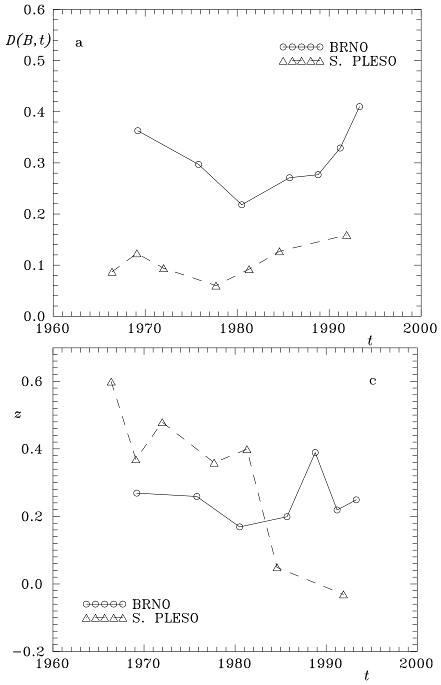
$$D(B, V) = 1.27K'(V) - 0.33E(B, t), \quad D(B, B) = K'(B) - E(B, t),$$

$$D(B, U) = 0.82K'(U) - 1.01E(B, t).$$

For the formulation of the above given relations we have exerted the following rules: when we have a complete set of measurements in $UBV$, we will use doubles $D(B, VB)$ and $D(B, VU)$, if there is no measurement in $B$, we will use $D(B, VU)$, if we have no $V$ extinction, we will use $D(B, c)$. This way we have found $N = 685$ entries on the dust component of extinction.

Basic information on the properties of the treated material is given in Tab. 3. Fig. 5b demonstrates that the distribution functions of the dust component of extinction are severally asymmetric and non-Gaussian. Moreover, the characteristics of them, namely the median $D(B, 1/2)$, the width $W$ and the asymmetry $z$ (see their definition in Paper I, Section 4) are time-independent.

Long-term variations of the characteristics of distribution functions are depicted in Fig. 7, where each of the points represents itself roughly one hundred measurements. In the beginning of the monitored time interval the median reached the value of $(0.52 \pm 0.08)$ mag/$X$. Then it decreased to a flat minimum in $1980.5 \pm 1.2$, when it diminished to its minimal value $(0.264 \pm 0.028)$ mag/$X$. After that, the median increased reaching $(0.40 \pm 0.04)$mag/$X$ at the end of the monitored region. The contemporary annual increase can be estimated as $(0.016 \pm 0.006)$ mag/$X$/year, i.e. appr. $(4.0 \pm 1.6)\%)$/year. The decrease of the width of the distribution function $W$ is very distinctive, and represented in the region covered by observations $(-0.014 \pm 0.003)$ mag/$X$/year, i.e. by $(-3.1 \pm 0.7)\%$ of the mean width per year. The decrease of the width of the distribution function stopped in $1986 \pm 4$ on $W = (0.326 \pm 0.027)$ mag/$X$. During the monitored interval the dust distribution function described by the parameter $z$ changed only insignificantly. The reasons for this evolution are unknown for us up to now.



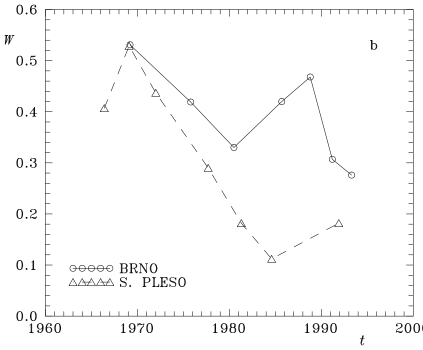




**Figure 7.** Long-term variations of the characteristics of distribution function of the dust extinction in the $B$ colour. Secular variations of **a)** the median of the dust extinction $D(B,t)$, **b)** the width of the distribution function $W(t)$, both in mag$/X$, **c)** its asymmetry $z(t)$, demonstrated on $\approx 90$ measurements of the dust extinction.

The most distinctive feature in seasonal variations is the dramatic drop of the level of dust extinction by a 1/3 of the original value, which takes place regularly in the first ten days of October. After this, the dust extinction more or less monotonously increasing until the next striking purification of the urban atmosphere. The jump in the course of seasonal variations of extinction can be found in all types of nights. The asymmetry $z$ and the width of distribution function $W$ does not seem to be affected by the jump. Attempts to find signs of this step in the course of seasonal variations of the particular extinction and the dust extinction at the Skalnaté Pleso Observatory have been unsuccessful.

The participation of seasonal and long-term variation on the total variability is roughly comparable. Wanting to deal with both variation scales simultaneously, we have constructed a quasiorthogonal model according to (37) comprising both time planes. Considering a possible dependence of the dust extinction on the level of solar activity, mentioned in Papoušek et al. (1984), we have introduced into the long-term part of the model an additional term, quantifying the solar activity by the so-called coronal index (Altrock et al., 1999).

The time dependence of the dust extinction has been expressed by a quasiorthogonal regression model with six free parameters:

$$D(B,t) = a_0\{1 + b_1(\tau - 0.654) + b_2[\sin(\pi\tau) + 0.613\tau - 1.006]+$$
$$b_3[CI(t) + 1.96\sin(\pi\tau) - 8.34\tau - 4.81]\}\{1 + B1[\sin(\pi T) - 0.530]$$
$$+B_2[T - 0.051\sin(\pi T) - 0.578]\}, \quad (38)$$

where: $\tau = (t - 62.57)/32$, $CI(t)$ – coronal index, $T = \mathrm{Frac}\ (t + 0.260)$. Reiteratively found parameters:

$$a_0 = (0.351 \pm 0.008)\mathrm{mag}/X,\ b_1 = -0.05 \pm 0.09,$$

$$b_2 = -0.53 \pm 0.09,\ b_3 = -0.00 \pm 0.05,\ B_1 = 0.08 \pm 0.07,\ B_2 = 0.37 \pm 0.08.$$

The standard deviation of one point is $s = 0.21$ mag$/X$, the mean uncertainty of fitting is 0.020 mag$/X$. The parameter $b_3$, expressing the measure of relationship between the dust extinction and the coronal index is practically zero, so we can conclude that solar activity does not have any measurable effect on dust extinction. For that reason we can omit the term $b_3$ in further calculations.[5]

The course of long-term variability of the dust extinction in $B$, corrected for seasonal variations is depicted in Fig. 8a, where the fitting function of secular variations is drawn. The dependence reached its maximum value at the very beginning of the reviewed time interval (1962), reaching $(157 \pm 11)\%$ of its arithmetic average. The function then decreases to its minimum in $(1980.9 \pm 1.2)$, when it dropped to $(84 \pm 3)\%$ of the average value. The extinction has increased again to $(119 \pm 5)\%$ in 1994.[6] We can estimate the rate of change of extinction at $(3.4 \pm 0.6)\%$/year of its immediate value.

The mentioned course of the dust extinction was probably caused by a real decrease of the concentration of dust particles of human origin. It took place because of the liquidation of dusty industrial workings and the transition from bad brown coal to liquid fuel and nuclear energy. the subsequent increase of extinction is apparently related to the increase of emissions and dustiness connected with the development of industrial production and traffic.

During the whole year, we observe monotonous increase of the dust extinction, which is interrupted by a sudden purification of the atmosphere around 5-th October (see Fig. 8b). Clearing of the atmosphere manifests itself in a quick decrease of the dust component of extinction, on average by $(37 \pm 8)\%$ which represents a drop by $(0.13 \pm 0.03)$ mag$/X$ in the $B$ colour. This unexpected phenomenon obviously typical for an urban observatory situated at a low altitude seems to be very persistent. It can be documented by the fact that the depths of the drop, corrected for secular trends in the first and the second halves of

[5]Because of the quasiorthogonality of the regression model, all the others remain unchanged.
[6]Detailed analysis of the course of the dependence reveals several statistically significant peaks based on a satisfactory quantity of measurements. Such prominent enhancements of extinction are from time to time the consequence of observational campaigns, when observations were carried out even in poor weather conditions. At the top, in the summer time a "string" of clear nights occurs sometimes, the extinction quality of them being gradually worse.

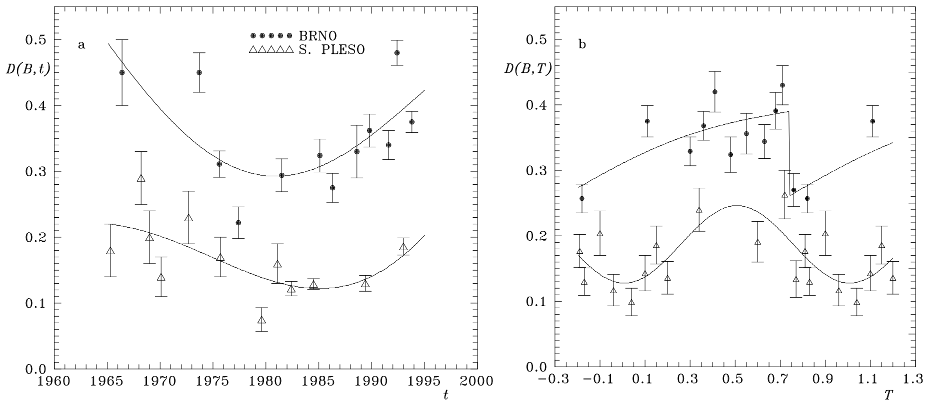



**Figure 8. a)** Long-term variability of the dust extinction in the $B$ colour corrected for seasonal variations. **b)** Seasonal variability of the dust extinction corrected for long-term trends. The fitting curves are computed using relations (38) and (39).

material were practically identical: $(0.12 \pm 0.04)$ mag/$X$, $(0.13 \pm 0.03)$ mag/$X$, respectively.

The nature of the jump in dustiness of the Brno atmosphere is somewhat unclear. A real decrease of convection streaming, which carries up the dust into the higher layers of atmosphere (Bednář 2000, Vaníček 2000) plays a role here. Further, we can speculate that in this period of year a radical change takes place in the meteorological conditions of the days with cloudless nights.

### 3.4. Variation of dust extinction – Skalnaté Pleso

The method of evaluation of the variable dust component is in the case of Skalnaté Pleso Observatory much more straightforward, as we use only a two-component extinction model:

$$D(B,V) = 1.261\,K'(V); \quad D(B,B) = K'(B); \quad D(B,U) = 0.823\,K'(U).$$

Basic information on the material is given in Tab. 3. The distribution function (see Fig. 5b) is strongly asymmetric, $z = 0.28$! The trends in the course of median, the width of distribution function and its asymmetry are demonstrated in Fig. 7. The course of the median is very similar to the course of the dust component in Brno, the mean value of extinction of a so called typical nights are two or three times smaller than in Brno. At the beginning of the pursued time interval the value of the median was $(0.109 \pm 0.014)$ mag/$X$. The median of the dust extinction then dropped to a flat minimum in 1978.5±2.9, when it descended to its minimal value: (0.093±0.010) mag/$X$. After that, it started to grow to (0.22±0.04) mag/$X$ at the end of the monitored time interval.

We establish (similarly as in Brno) a pronounced narrowing of the width of the distribution function: the mean rate is $(-0.011\pm0.004)$ mag/$X$/year.

This drop is dictated here chiefly by the improvement of the accuracy of the extinction measurements (the value of uncertainty of measurements in the very beginning of the observations is comparable with the observed width of the distribution function), subjective factors were significant, too. It is documented expressively by the evolution of the asymmetry of the distribution function $z$, which steadily decreased from its initial value $z = 0.51 \pm 0.08$ to a quite opposite (and physically hardly understandable) value: $z = -0.33 \pm 0.22$! Such a non-standard development is probably a consequence of growing criticism by the observers (technicians) of non-perfect nights, which are then missing. Therefore, we must be extremely cautious in interpretation of the results connected with the long-term variations of extinction.

The observed seasonal variations of the median, the width of distribution function and its asymmetry are sinusoidal. The maximum of median occurs in times of summer dusty nights, when the observatory stands below the inversion level. The distribution function exhibits the maximum width in summer, when there are the largest differences among good and poor observational nights. In winter times, when the inversion layer is below the observatory, the quality of all nights is practically identically good.

The time variability of the dust extinction was described by a sixparametric quasiorthogonal model dealing with long-term changes, variations dependent on the coronal index $CI(t)$ and seasonal variations:

$$D(B,t) = a_0\{1 + b_1[\cos(\pi\tau) - 0.148] + b_2[\cos(\pi/2\tau) - 0.401\cos(\pi\tau) - 0.649] + b_3[CI(t) - 5.78\cos(\pi/2\tau) + 3.18\cos(\pi\tau) - 6.30]\}\{1 + B_1[\cos(2\pi T) - 0.352] + B_2[\sin(2\pi T) + 0.079]\}, \quad (39)$$

where $\tau = (t - 63.08)/32.82$, $T = t + 0.033$, and the parameters found:

$$a_0 = (0.168 \pm 0.008)\mathrm{mag}/X,\ b_1 = 0.24 \pm 0.08,$$

$$b_2 = -2.3 \pm 0.9,\ b_3 = 0.008 \pm 0.012,\ B_1 = -0.34 \pm 0.09,\ B_2 = -0.09 \pm 0.07.$$

As in Brno, the correlation between the dust extinction and the coronal index is dubious, so we can omit the term with $b_3$. All the other input values of parameters and their errors remain valid. The mean value of the dust extinction $(0.186 \pm 0.011)$ mag/$X$ is significantly larger than the arithmetic average, because the majority of the observations were done in winter, when the extinction was minimal. The standard deviation of one measurement is relatively large: $s = 0.227$ mag/$X$. The participation of seasonal and long-term variation on the total variability are approximately comparable.

In the course of secular variations we observe a decrease from the initial value of (132±8)% to a local minimum in (1985.1±1.0) to (72±8)% of the arithmetic average. Then an increase developed to (129±24)% at the very end of the monitored time interval. The minimum of the dust extinction occurs in the very beginning of the year: (3rd Jan.± 10 d), which corresponds to the times of the deepest inversion. The time course of both long-term and seasonal variation of the dust extinction is depicted on Fig. 8.

## 4. Comparison of observations with the model. Prediction of the further course of extinction

The knowledge of long-term variations of extinction in the $B$ colour and the conversion relations (see Tab. 4) among the extinction coefficients in individual colours enables us to predict the behaviour of extinction in the region covered by observations and in adjacent regions. For the Rayleigh extinction $R(c)$ the validity of relations (4) to (9) has been supposed. The conversion relations among extinction coefficients relating to the dust component are the consequence of the amenability of power law with the parameter $\alpha = 1.04 \pm 0.04$. Mutual conversions of particular components in the individual colours follow from assumption (28) supposing the occurrence of it only in the colours $U, B$ and $V$.

The validity of the proposed models of atmospheric extinction in Brno and Skalnaté Pleso Observatories will be tested by comparison of the long-term and seasonal variations of the observed extinction coefficients in all colours of the *UBV* system with the model prediction. Within the framework of the accepted extinction models, we then discuss the expected course of mean extinction in the colours of the *UBVRI* photometric system in which the majority of photometric measurements is performed.

**Table 4.** Conversion relations among the extinction coefficients

| Colour | $\lambda_{\rm eff}$ | $[D(c)/D(B)]$ | $[E(c)/E(B)]$ | $R(c,h)$ – Brno | $R(c,h)$ – S. Pl. |
|---|---|---|---|---|---|
| $U$ | 365 | 1.214±0.009 | 1.24±0.07 | 0.556 | 0.463 |
| $B$ | 440 | 1 | 1 | 0.257 | 0.213 |
| $V$ | 550 | 0.793±0.007 | 0.26±0.09 | 0.134 | 0.116 |
| $R$ | 700 | 0.617±0.012 | 0 | 0.039 | 0.033 |
| $I$ | 880 | 0.486±0.014 | 0 | 0.015 | 0.013 |

### 4.1. Comparison of the observed $V$, $B$ and $U$ extinctions with the model prediction

In the case of the Skalnaté Pleso Observatory we get a simple linear two-component model, in which the resultant extinction coefficient is the sum of a constant (Rayleigh) contribution and a time-dependent dust component in pertinent colours. The time-dependency of the mean value of the dust extinction $D(B,t)$ can be represented by the relation (39).

The comparison between extinction model predictions and observations for long-term variations in *UBV* colours is displayed on Fig. 9a, while Fig. 9b represents the same for seasonal variations corrected for long-term trends. It seems the average values of differences between the observed and calculated values of the extinction coefficients in all three colours are smaller than the errors of their determination. We state the suggested two-component model of atmospheric extinction corresponds to the observations obtained at the Skalnaté Pleso Observatory excellently.

In the case of the Brno Observatory we has been forced to use a three-component model, in which the resultant extinction is the sum of the constant (Rayleigh) extinction in the relevant colour and variable dust and particular extinctions according to the relation (18). The time dependencies of the mean value of the dust extinction $D(B,t)$ and the particular extinction $E(B,t)$ have been assumed in the forms of (38) and (37), respectively. The comparison of the three-component model with the observations represented by the so called normal points with their internal errors in the $U, B$ and $V$ colours for long-term variations (corrected for seasonal variations) is demonstrated in Fig. 9c. Seasonal variations corrected for long-term trends are presented in Fig. 9d. Even here the average values of the differences between the observed and calculated values of the extinction coefficients are comparable with the errors of their determination. We could establish that the proposed model of atmospheric extinction fits the observations done at the urban Brno Observatory quite well.

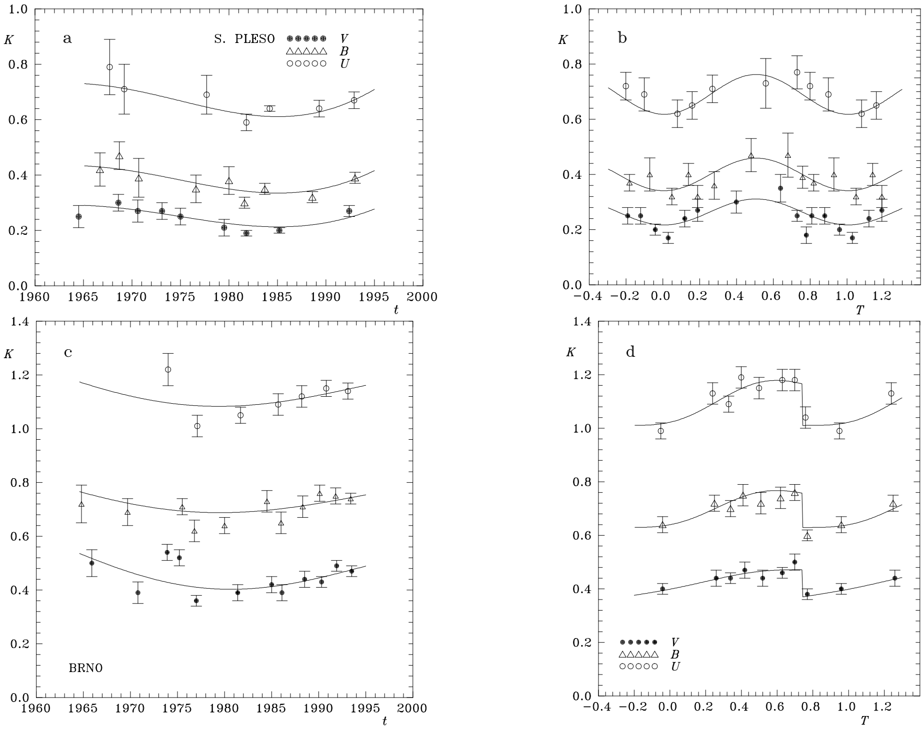



**Figure 9.** Long-term variations of the coefficient of extinction $K$ in the $UBV$ system with the model for Skalnaté Pleso **(a)** and Brno **(c)**, seasonal variations, corrected for long-term trends for Skalnaté Pleso **(b)** and Brno **(d)**.

Tab. 5 provides interesting information: calculated mean values of the extinction coefficients in individual colours of the photometric *UBVRI* system, together with the percentage terms of individual components of extinction. The

**Table 5.** Contribution of the individual components of extinction in the whole time interval studied.

| | Brno | | | | Sk. Pleso | | |
|---|---|---|---|---|---|---|---|
| Colour | $K$[mag] | $\%R$ | $\%D$ | $\%E$ | $K$[mag] | $\%R$ | $\%D$ |
| $U$ | 1.10 | 51 | 39 | 10 | 0.69 | 67 | 33 |
| $B$ | 0.70 | 37 | 50 | 13 | 0.40 | 53 | 47 |
| $V$ | 0.44 | 31 | 64 | 5 | 0.26 | 44 | 56 |
| $R$ | 0.26 | 15 | 85 | | 0.15 | 22 | 78 |
| $I$ | 0.19 | 8 | 92 | | 0.10 | 13 | 87 |

value of the mean extinction rises very steeply with decreasing wavelength of light, which is the natural consequence of the wavelength behaviour of all the considered types of extinction.

The constant - the Rayleigh component plays a decisive role only in the $U$ and $B$ colours for the Skalnaté Pleso Observatory. The maximum influence of the Brno particular extinction (about 13%) is noticed in the $B$, it is significant in the $U$ colour, too. The dust extinction has a predominant position among the other types of extinction in all other colours, it noticeably prevails namely in the longer region of spectrum, where the atmosphere itself does not significantly interfere in the passage of the light of cosmic objects.

### 4.2. Prediction of the development of extinction in the *UBVRI* system in the near future

The relatively good agreement of the proposed time-dependent extinction models with the observations and the monotonous course of extinction at the end of the studied time interval entitles us to make certain prediction concerning extinction in the near future. It seems probable that the dust component at both stations will continue to grow, while the Brno particular extinction will on the contrary decrease. The future course of the time dependencies has been approximated by tangents to the long-term dependencies in terminal points of the interval covered by the observation.

**Table 6.** Prediction of the development of the mean components of extinction.

| | Brno | | | Skalnaté Pleso | | |
|---|---|---|---|---|---|---|
| | 1995 | 2000 | 2005 | 1995 | 2000 | 2005 |
| $U$ | 0.71±0.05 | 0.82±0.06 | 0.93±0.10 | 1.164±0.033 | 1.21±0.05 | 1.27±0.07 |
| $B$ | 0.41±0.04 | 0.51±0.05 | 0.60±0.08 | 0.757±0.027 | 0.79±0.04 | 0.84±0.05 |
| $V$ | 0.28±0.03 | 0.35±0.04 | 0.43±0.06 | 0.490±0.020 | 0.537±0.029 | 0.59±0.04 |
| $R$ | 0.160±0.023 | 0.22±0.03 | 0.28±0.05 | 0.301±0.015 | 0.345±0.022 | 0.39±0.03 |
| $I$ | 0.114±0.018 | 0.162±0.026 | 0.21±0.04 | 0.221±0.013 | 0.256±0.018 | 0.290±0.025 |

The prediction of the mean value of the extinction in the *UBVRI* system is given in Tab. 6. The numbers are very expressive and indicate that it would be wise to switch to photometric observations done in the region of longer wavelengths, where the atmospheric extinction does not play such a severe role.

## 5. Conclusions

**1.** All the conclusions concerning the properties of the atmospheric extinction have been done on the basis of the thorough treatment of two extensive sets of 1779 measurements of the *UBV* extinction coefficients measured in 731 individual nights in the last 30 years at two observatories with very different observational conditions. The detailed specification of the observational material obtained at Brno and Skalnaté Pleso Observatories is given in Paper I.

**2.** For the interpretation of observations from the Skalnaté Pleso Observatory a linear two-component model of atmospheric extinction fits well. The more or less constant component of the model is determined namely by the Rayleigh scattering on clumps of molecules of air $R(c)$, for which the equation $R(c) \sim \lambda_c^{-4} e^{-h/7996\,\mathrm{m}}$ is valid. $\lambda_c$ is the effective wavelength of the particular colour and $h$ is the altitude of the observational station. Due to the higher altitude of the Skalnaté Pleso Observatory, the Rayleigh contribution to the extinction is there 20% smaller than in Brno. The variable component of the model clearly corresponds to the scatter of light on dust particles - aerosols $D(c)$.

Attempts to apply such a simple model to the Brno observations were unsuccessful. We were forced to expand the above mentioned two-component model of atmospheric extinction with a further independent variable component $E(c)$, arbitrarily called as *particular extinction.*

**3.** From the slopes of the relations among the extinction coefficients, obtained simultaneously in different colours in both observatories, a spectral parameter $\alpha$ has been derived, describing the extinction caused by the scatter of light on dust $D(c)$ by the relation $D(c) \sim \lambda_c^{-\alpha}$, where $\alpha = 1.04 \pm 0.04$. This fact indicates the dust extinction is caused namely by classical Mie's scattering on dielectric particles of diameters comparable to the wavelength of the light.

The mean dust extinction in Brno is in the interval studied 84% greater than at Skalnaté Pleso. The average contributions of the dust extinction to the total extinction are at both stations comparable to the contributions of the Rayleigh scattering. In Brno in $U, B, V$ the shares of the dust extinction on the total extinction are 39%, 50% and 64%, respectively. At the Skalnaté Pleso Observatory the influence of the dust extinction is a bit smaller: 33% in $U$, 47% in $B$ and 56% in $V$.

**4.** The power law is invalid for the particular extinction $E(c)$. The extinction mechanism probably will be an absorption in spectral bands of so far unidentified compounds located mainly in the region of the $B$ and $U$ colours. The transformation relations among the contributions of the particular extinction in individual colours have been estimated. The mean contribution of the particular extinction on the total atmospheric extinction in Brno was the most significant in $B$ – 13%, in $U$ and $V$ colours 10% and 5%, respectively.

**5.** By means of the model of atmospheric extinction, values of its variable components in $B$ were calculated for both observatories. This material helps us to study the secular and seasonal variations of characteristics of the distribution functions both for the particular and the dust extinction, as well as variations of their mean values. Long-term and seasonal variations of the mean values of extinctions have been found as mutually independent modulations of a certain mean value of the relevant extinction, represented by its arithmetic average.

Seasonal variations of the dust component of extinction on the pair of observatories studied, exhibit different courses. The Brno dust extinction reaches its minimum after October (5±4). The cause of the unforeseen clean up of the Brno atmosphere represented by a sudden decline of the extinction by (37±8)% remains unclear up to now. The drop of extinction is then followed by a slow monotonous increase, ended by the next severe shrinkage of the dust extinction.

Seasonal variations of the dust extinction at the Skalnaté Pleso Observatory seem to be controlled by the annual variations in the position of the layer of the temperature inversion. The atmosphere is very dusty under this layer and relatively very clear above it. In summer time, the inversion frequently ascends above the observatory, which is the main cause of the observed more or less sinusoidal variations of extinction with the minimum around January(3±10). At that time, the dust extinction decreases to (76±6)% of its average value and a half-year earlier it climbs to (146±12)%.

Systematic measurements of the concentration of aerosols in the Earth atmosphere were conducted by The Solar and Ozone Laboratory of the Czech Hydrometeorological Institute in Hradec Králové. The seasonal course of the aerosol component of extinction measured through the whole solar spectrum (the global insolation) was more or less sinusoidal with a summer maximum (120% of the annual average) and a winter minimum (80%). According to Vaníček (2000) the observed variations are the results of both the annual course of convective streaming in the atmosphere, transporting into the air particles of soil aerosols, in particular and seasonal changes of the character of the Earth's ground (e.g. snow cover in winter). If there are materials of industrial and traffic origin among the sources of the dust particles of aerosols too, the course of seasonal variations could be quite different.

**6.** The observed seasonal variation of the particular extinction exhibits a sinusoidal rundown. The maximum of it ((116±5)% of the average) occurs on July (16±13), the minimum (64±11)% a half-year later. The curve of variation of this component of extinction very truly copies the curve of seasonal variations of the average temperature of the ground layers of the air.

The behaviour of the particular extinction indicates it ccould be a consequence of the absorption of light on the particles of ground layers smog, optically activated by specific photochemical reactions.

**7.** The majority of extinction observations at the Skalnaté Pleso were done from October to March (Paper I, Fig. 1b), when the contribution of the dust extinction is minimal, while the preponderance of observationally exploitable nights in Brno falls to the time interval May – October, when the dust extinction culminates. The only exception to the rule is October, the month at the beginning of which an extreme clean up of dust particles takes place, and the influence of the particular extinction fades, too. The percentage of clear nights is satisfying, furthermore they are relatively long.

**8.** The distribution function of the particular extinction seems to be symmetric and its characteristics do not exhibit any seasonal or long-term changes. The particular extinction (see Fig. 6a) appeared in the Brno atmosphere in the mid-sixties, reached its flat maximum ((125±5)% of its arithmetic average) at (1981.9±0.6). At the end of the time interval covered by the observations in 1994, it dropped to (70±8)% of the average value. Now it is probably decreasing.

**9.** We see a very strong decrease of the width of the distribution function $W$ of the Brno dust component (see Fig. 7b), which decreased in the years 1962–1994 to 45% of its initial value. The decline in question continued until 1986±4, when it stopped at the value: $W = (0.326 \pm 0.027)$ mag/$X$. The observed phenomenon is apparently real, but the cause of it remains unknown.

Characteristics of the distribution function of the dust extinction at the Skalnaté Pleso Observatory exhibit still more dramatic evolution (see Fig. 7b, c). We see here a steady decrease of the its width $W$, which is dictated primarily by the improvement in accuracy of determination of the individual extinction coefficients. We have to remark that the standard error of a measurement in the very beginning of the observations was for a long time comparable with the observed halfwidth of the distribution function. At the peak, we witness the growing strictness in selection of observational nights, which can be documented by a monotonous decline of the asymmetry of the distribution function $z$ to its final physically unreal negative value: $z = -0.33 \pm 0.22$! These facts diminish the credibility of the observational material obtained at the Skalnaté Pleso Observatory and force us to be cautious in its interpretation.

**10.** In the course of the long-term variations of the mean value of the dust extinction corrected for seasonal effects, obtained at the Skalnaté Pleso we see a decrease from the initial value ((132±8)% of the average value) to a local minimum in 1985 to (73±7)%. Then it icreased to (129±24)% at the end of the time interval studied (see Fig. 8a). The current rate of the relative growth of the dust extinction is (11±4)%/year.

Analogous progress can be seen in Brno, where the mean dust extinction was largest in the beginning of the observations in 1962, reaching (157±11)% of its arithmetic average. Then a decline followed to a minimum in 1981, when the dust extinction decreased to (84±3)% of the average value. Since 1981 the extinction started to grow again and reached (119±5)% in 1994. We can speculate that the mean value of the dust extinction is increasing now with the rate of (4.0±0.7)%/year.The absolute rates of the increase of the dust extinction at

both observatories are nearly identical. The initial decrease of the dust extinction to its minimum in eighties can be interpreted as a positive consequence of transition to environmentally better (gas or liquid) type of fuel. The subsequent increase was very probably caused by further extensive development of industry, motoring and other sources of air pollution.
**11.** The mean dust extinction averaged over the whole time interval studied in the $B$ colour for Brno is $(0.354{\pm}0.011)$ mag/$X$, and for Skalnaté Pleso is $(0.186{\pm}0.011)$ mag/air mass. Their ratio $r = (0.525 \pm 0.035)$ is the same for the other two colours, too. Supposing a quasistatic situation, when the concentration of the dust particles decreases exponentially with the altitude of the observational station $h$, then: $D(c,h) \sim e^{-h/H_{\rm D}}$, where $H_{\rm D}$ is the so called "dust scale height". Using the ratio $r$ and the altitudes of our observational sites, we can estimate the dust scale height: $H_{\rm D} = (2.29 \pm 0.24)$ km. This value is rather greater than those denoted in literature (e. g. Hayes & Lantham, 1975 give: $H_{\rm D} = 1.5$ km). Nevertheless, we should realize that both observatories are realtively distant and so the sources of dust particles are probably different. The relatively greater dustiness (greater $H_{\rm D}$) of the atmosphere above the Skalnaté Pleso could be explained by the fact that there are very active vertical air streaming there, which drag the dust particles to higher atmospheric layers. Applying formally the same model conception for the stratification of the hypothetical particles causing particular extinction: $E(c,h) \sim e^{-h/H_{\rm P}}$, which is in the framework of errors of the observations non observable at Skalnaté Pleso, we could at least assess certain upper limit of the "particular scale height": $H_{\rm P} < 0.8$ km. Really we must take into our consideration not only the altitudes, but the position of the observatory in respect to pertinent sources of pollution, too.
**12.** The correlation between the value of the dust extinction at both observatories and the coronal index, quantifiing the solar activity has not been proven. The analogous correlation between the particular extinction and the coronal index has not been discussed by reason of the small relative accuracy of data.
**13.** The comparison of the accepted models of the atmospheric extinction with the observed extinction coefficients obtained for all three colours of the $UBV$ system on both seasonal and long-term scales establishes that the models describe the observed reality satisfactorily well. This fact entitle us to predict the future trends of extinction on both observatories in all colours of the extended Johnson's $UBVRI$ photometric system.

For as much the dust extinction at both localities will very probably grow in the near future, our models (see Tab. 6) predict that the increase should not mean an exit of photometric works at both observatories. Nevertheless, it is worthwhile to premeditate whether it is inevitable in stellar photometry to adhere to the historically originated $UBV$ system. We conclude that observations namely in the $U$ colour becomes more and more badly affected by the growing extinction. Nevertheless, it is not reasonable to quit the near ultraviolet measurements when studying hot stars, as these measurements inform us about the astrophysically crucial region behind the Balmer jump.

At the Brno Observatory we were compelled to discard measurements in $U$ several years ago, not only for the too high extinction but also because of the replacement of a light detector. The currently used CCD camera is much less sensitive in the ultraviolet part of the spectrum than a classical photomultiplier. That is why we now use the broadband $BVR$, pertinently the $VRI$ photometric system. In these spectral regions the Brno atmosphere is quite transparent and the extinction itself does not present a serious problem.

Problems with the near future extinction at the Skalnaté Pleso Observatory thanks to its high altitude are not very topical.

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