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About two possible dynamical evolution ways of extra-solar planet γ Cephei

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Abstract. Two possible ways of dynamical evolution of the extra-solar planet γ Cephei are studied in dependence on the mutual orientation of the planet and the distant star orbits. The Lyapunov stability of the planet orbit is investigated. The evolution of the orbits when resonances are absent is described by the Hamiltonian without short-periodic terms, eliminated by von Zeipel's method.

The question about Lyapunov's stability of the extra-solar planets is considered in the frame of the general three-body problem, i.e. a planet in a binary system revolves around one of the component, and the distance between the star components is much greather than that between the orbiting star and the planet. The differential equations with regards to the eccentricity and the argument of the perigee of the planet, with this Hamiltonian allow to find conditions of the stability for $t > t_0$. The possible conditions of the stability of the extra-solar planet are presented by its orbital parameters – the angle of the mutual inclination between the planet and the distant star orbits and the angular momentum of the system.

The initial conditions for the investigation of the dynamical evolution were taken from two different sources. In the first case from data of Hatzes et al. (2003) and Schneider (2005), in the second case from Neuhäuser et al. (2007).

For the first case we obtained the conditions of the stability of the planetary orbit for some values of the longitude of the node laying in defined limits and the value of the mutual inclination. In the second case the angle of the mutual inclination has such a value that small deviations of the initial elements may become in future large for all values of the node. In this case the motion is unstable with respect to the eccentricity of the planetary orbit e_1 (Chetaev, 1965). The results of the theory were verified by numerical integration.

Key words: three-body problem – extra-solar planet – dynamical stability – γ Cephei

1. Introduction

The present paper continues our investigation of the dynamical stability and behavior of extra-solar planets in binary stellar systems, wherein the ratio of the semi-major axes of orbits of a planet and of a distant star is a small parameter

(Solovaya and Pittich, 2004). The number of known binary or triple systems with planets is at least ten binary and five triple systems with Jupiter-like planets (Musielak et al., 2005). There are several analytical and numerical studies in the area of the dynamical stability of orbits. We remind that we understand the stability as the conservation of the configuration of the system over an astronomically long time interval, when small deviations of the initial conditions do not become large in the future.

The eccentricity of the planetary orbit and the mutual inclination of the orbits change in small limits, and there are no close approaches between bodies which could lead to the destruction of the system. The motion is considered in Jacobi's coordinate system and the invariant plane is taken as the reference plane. For description of the evolution we used the Keplerian osculating elements and the canonical Delaunay elements L_j, G_j, and g_j ($j = 1$, for the planet orbit, $j = 2$ for the distant star orbit). The canonical Delaunay elements can be expressed through the Keplerian elements as

$$L_j = \beta_j \sqrt{a_j}, \qquad G_j = L_j \sqrt{1 - e_j^2}, \qquad g_j = \omega_j \,, \tag{1}$$

where

$$\beta_1 = k \frac{m_0 m_1}{\sqrt{m_0 + m_1}}, \qquad \beta_2 = k \frac{(m_0 + m_1)\, m_2}{\sqrt{m_0 + m_1 + m_2}} \,. \tag{2}$$

m_0, m_2 – the masses of the stars, m_1 – the mass of the planet, k – the Gaussian constant, a_j – the semi-major axis, e_j – the eccentricity, and ω_j – the argument of the perigee.

We used the Hamiltonian of the system without the short-periodic terms from the paper of Solovaya and Pittich (2004, formula (3)) and retained all symbols of this work. Possible conditions of the stability are presented by the following parameters: cosine of the angle of mutual inclination q between the planet and the distant star orbits

$$q \; = \; \frac{c^2 - G_1^2 - G_2^2}{2 G_1 G_2}, \tag{3}$$

and the angular momentum of system c, which is constant. The eccentricity of the distant star orbit is constant too and the eccentricity of the planet orbit can change in a large interval.

Applying the derived conditions of the stability (Solovaya and Pittich, 2004) to the system γ Cephei we will show two different ways of the evolution of this system in dependence on the initial conditions – on the mutual orientation of the orbits of the distant star and the planet, and on the stability or unstability of the planetary orbit with respect to its eccentricity.

For our investigation it is necessary to know the six pairs of the initial values of the Keplerian elements and masses of all components. In the first case we used the masses and the elements from the papers of Hatzes, et al. (2003) and Schneider (2005). In the second case we used there data from the paper of Neuhăuser et al. (2007). In both cases the elements of the planetary orbit have different values of the angular elements and a very different value of the planet's mass. We obtained two different ways of the dynamical evolution of the investigated system, which will be illustrated in Figures.

2. Orbit of the planet

The γ Cephei presents a very interesting and suitable system for application to our theory. This system is known as a spectroscopic triple system located at the heliocentric distance of 13.8 pc, composed of a primary star, a stellar-mass companion in 70 year orbit, and a substellar companion that is a planet. Fuhrmann (2004) estimates the primary mass of γ Cephei as $1.59\,\mathrm{m}_\odot$ and the minimum mass of the planet as $1.76\,\mathrm{m}_J$. Torres (2007) estimates the upper mass limit of the planet between $17-19\,\mathrm{m}_J$.

According to our theory the eccentricity of the distant stellar companion is constant, but the eccentricity of the planet can change in a large interval depending on the angle of the mutual inclination of the orbits of the planet and the distant star. The eccentricity is defined by the formula $e_1 = \sqrt{1-\xi}$, the maximum and the minimum value of the eccentricity change between ξ_1 and ξ_2: $e_{1max} = \sqrt{1-\xi_1}$ and $e_{1min} = \sqrt{1-\xi_2}$. There are only two roots, which are less than unity and their meaning ξ_1 and ξ_2 can be defined from equations

$$f_2(\xi) = \xi^2 - 2\left(\bar{c}^2 + 3\,\overline{G}_2^2\right)\xi + \left(\bar{c}^2 - \overline{G}_2^2\right)^2 + \frac{2}{3}\,(10 + A_3)\,\overline{G}_2^2 = 0\,, \quad (4)$$

$$\begin{aligned} f_3(\xi) = {} & \xi^3 - \left(2\,\bar{c}^2 + \overline{G}_2^2 + \frac{5}{4}\right)\xi^2 + \\ & + \left[\frac{5}{2}\left(\bar{c}^2 + \overline{G}_2^2\right) + \left(\bar{c}^2 - \overline{G}_2^2\right)^2 - \frac{1}{6}\,\overline{G}_2^2\,(10 + A_3)\right] - \\ & - \frac{5}{4}\left(\bar{c}_2 - \overline{G}_2^2\right)^2 = 0\,, \end{aligned} \quad (5)$$

where

$$A_3 = 2 - 6\,\eta_0^2\,q_0^2 - 6\,\left(1-\eta_0^2\right)\left[2 - 5\,\left(1 - q_0^2\right)\sin^2 g_{1_0}\right]\,. \quad (6)$$

$\overline{G}_2 = G_2/L_1$, $\bar{c} = c/L_1$. The subscript or superscript 0 denotes starting values of all parameters.

In the first case we used the masses of components $m_0 = 1.59\,\mathrm{m}_\odot$, $m_1 = 1.76\,\mathrm{m_J}$ and $m_2 = 0.4\,\mathrm{m}_\odot$. The orbital elements of the planet (index 1) and the distant star (index 2) are following (Hatzes et al, 2003; Schneider, 2005):

$$
\begin{aligned}
m_1 \sin i_1 &= 1.59\,\mathrm{m_J}, & & \\
e_1 &= 0.12, & e_2 &= 0.361, \\
a_1 &= 2.13 \text{ AU}, & a_2 &= 18.5 \text{ AU}, \\
i_1 &= 64.6^\circ, & i_2 &= 51.3^\circ, \\
\omega_1 &= 49.6^\circ, & \omega_2 &= 158.76^\circ, \\
\Omega_1 &= 75.6^\circ, & & \\
T_1 &= 2\,453\,121.925 \text{ JD}, & T_2 &= 2\,448\,429.03 \text{ JD}.
\end{aligned}
$$

Because the longitude of the ascending node of the distant star is unknown we change its value from 0° till 360°. So as we consider the motion in the Jacobian coordinate system and the invariable plane is the reference plane, the angular elements of the system must be transformed into the elements to the invariable plane. For such initial data we obtained next general constants: the angular momentum of system $c = 12.19$ AU$^2\,\mathrm{m_\odot}\,\mathrm{yr}^{-1}$, $\overline{c} = 627.89$, and $\overline{G}_2 = 627.45$. For the initial elements the variation of the cosine of the angle of the mutual inclination I, q, as a function of the value of the longitude of the ascending node Ω_2 of the secondary star from the interval $0^\circ - 360^\circ$ is presented in Figure 1. In Figure 2 theoretical minimum and maximum values of the mutual inclination I, in Figure 3 theoretical minimum and maximum values of the eccentricity e_1 of the planet are plotted. Within the interval $31^\circ < \Omega_2 < 120^\circ$ the planetary orbit is stable, outside this interval is unstable.

According to our theory (Solovaya and Pittich, 2004), when cosine of the mutual inclination $q_0 < q_{01}$ or $q_0 > q_{02}$, then ξ, which is less than 1, is the smallest root of a third degree equation. The maximum value of the eccentricity of the planet orbits can exceed the initial value of the eccentricity only on value of the short-periodic perturbations. When $q_{01} < q_0 < q_{02}$, the eccentricity of planet orbit can increase.

The eccentricity of the planet is a small value. Therefore we can also use the criterion of stability obtained for the near circular orbits (Solovaya and Pittich, 2004), determining the sign of the constant B.

Constant B is positive when value of Ω_2 is in the interval from 31° to 120°. The eccentricity change within this interval a little. It means that the motion is stable with respect to e_1 (Chetaev, 1965). For all other values of the node the value B is negative and small deviations of the initial elements may become large in the future.

So for $\Omega_2 = 180^\circ$ the angle of mutual inclination of orbits $I = 84.628^\circ$. The eccentricity changes for these values of Ω_2 and I within the interval $0.117 < e_1 < 0.993$. At the maximum value $e_{1_{max}} = 0.993$ the minimum perigee distance $r_{T_1} = 0.015$ AU. Fuhrmann (2004) estimates the radius of the primary star of $R = 4.66\,\mathrm{R_\odot}$ (the solar radius $\mathrm{R_\odot} = 0.00465$ AU). In such case the planet in its perihelion is below the primary star's surface.

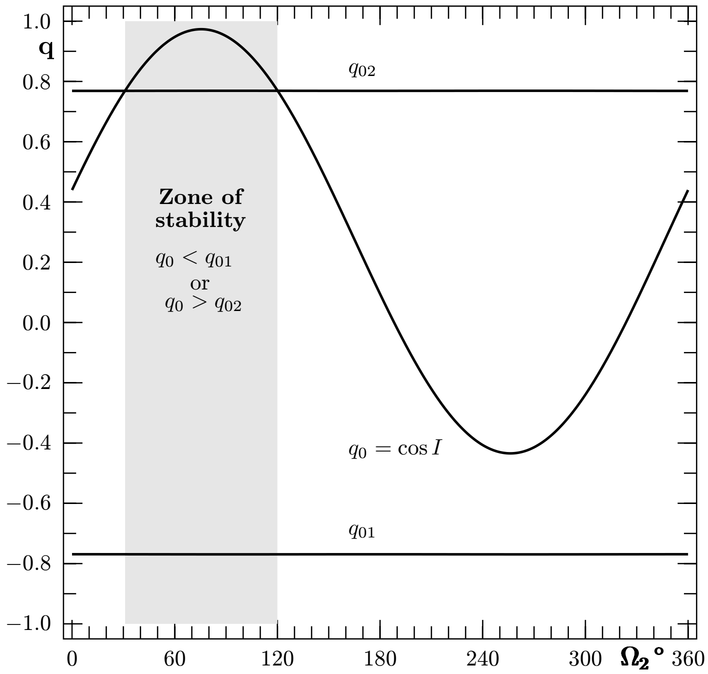

Figure 1. Theoretical boundaries of the stable zone. The stable motion of the planet of γ Cephei is limited with values of the longitude of ascending node of the secondary star from the interval $31^\circ < \Omega_2 < 120^\circ$.

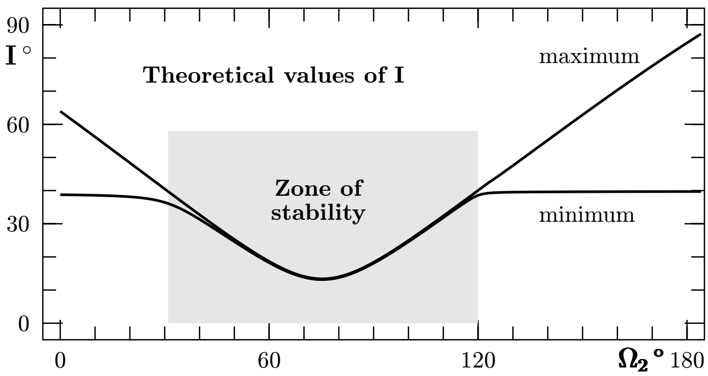

Figure 2. Theoretical minimum and maximum values of the mutual inclination I versus values of the longitude of ascending node of the secondary star from the interval $0^\circ < \Omega_2 < 180^\circ$.

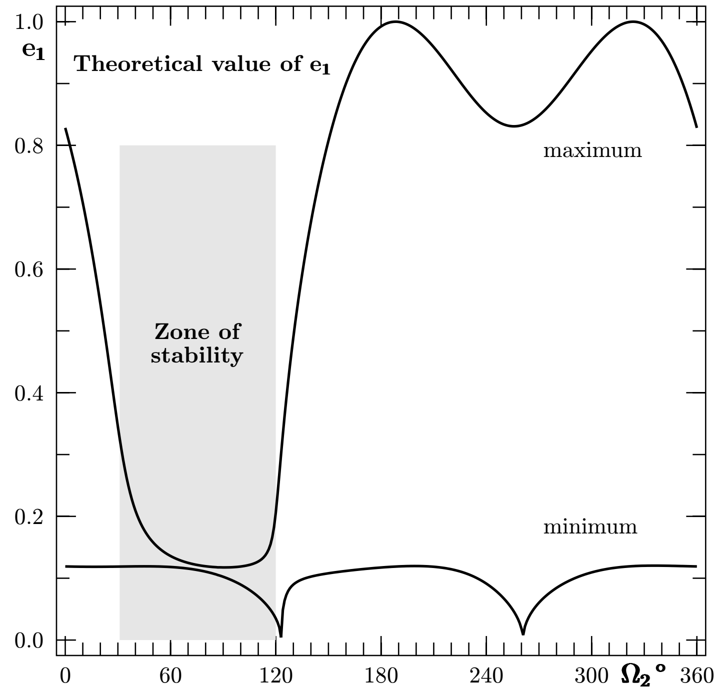

Figure 3. Theoretical minimum and maximum values of the eccentricity e_1 of the planet versus values of the longitude of ascending node of the secondary star from the interval $0° < \Omega_2 < 360°$.

The comparison of results with the results obtained by the numerical integration (see Figure 4) showed that the used analytical method gives good results. The boundary of gray zones were computed from the theory. The doted curves are results of numerical integration. We used results of the numerical integration for the comparison with results of our theory for the interval of 50,000 years, but for better illustration this interval is displayed in Figure 4 for only 5,000 years. In Figure 4 we can see that the periodicity of changes of orbital elements do not exceed the minimum and maximum values obtained from the theory.

In the second case we took the elements from the paper of Neuhăuser et al. (2007). By combining the radial velocity, astrometric and imaging data authors have refined the binary orbit and determined the dynamical masses of the two stars in the γ Cephei system. Namely masses of the components are $m_0 = 1.4\,\mathrm{m}_\odot$, $m_1 = 17\,\mathrm{m_J}$, and $m_2 = 0.409\,\mathrm{m}_\odot$. The orbital elements of the planet and the distant star are following:

$$m_1 \sin i_1 = 1.60\,\mathrm{m_J},$$

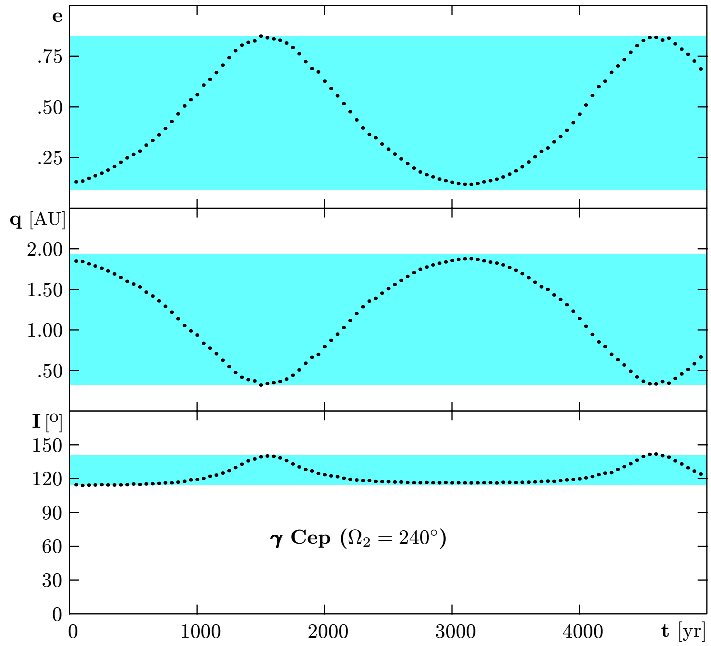

Figure 4. Orbital evolution of the planet of γ Cephei within the interval of 5000 years covering several revolutions of the distant star for $\Omega_2 = 240°$. Dots represent results obtained by the numerical integration. The gray zones are the boundaries of the orbital elements computed by the analytical theory.

$$
\begin{aligned}
e_1 &= 0.115, & e_2 &= 0.4112,\\
a_1 &= 2.044 \text{ AU}, & a_2 &= 20.18 \text{ AU},\\
i_1 &= 5.40°, & i_2 &= 119.3°,\\
\omega_1 &= 63°, & \omega_2 &= 161.01°,\\
 & & \Omega_2 &= 18.04°,\\
T_1 &= 2\,453\,147\,\text{JD}, & T_2 &= 1991.605\,yr.
\end{aligned}
$$

From these elements the following general constants were computed: The angular momentum of the system $c = 10.95$ $\text{AU}^2\,\text{m}_\odot\,\text{yr}^{-1}$, $\overline{c} = 63.86$, and the parameter $\overline{G}_2 = 64.31$. We see that the parameters $\overline{c}$ and $\overline{G}_2$ are 10 times smaller than in the first case. Because the longitude of the ascending node of the orbit of the planet is unknown we changed the value of Ω_1 from 0° to 360°.

For these initial elements the variation of the cosine of the angle of the mutual inclination I, q, as a function of the value of the longitude of the ascending node Ω_1 of the primary star from the interval $0° - 360°$ is presented in Figure 5. In this case the cosine of the mutual inclination q_0 for any values of Ω_1 is always inside

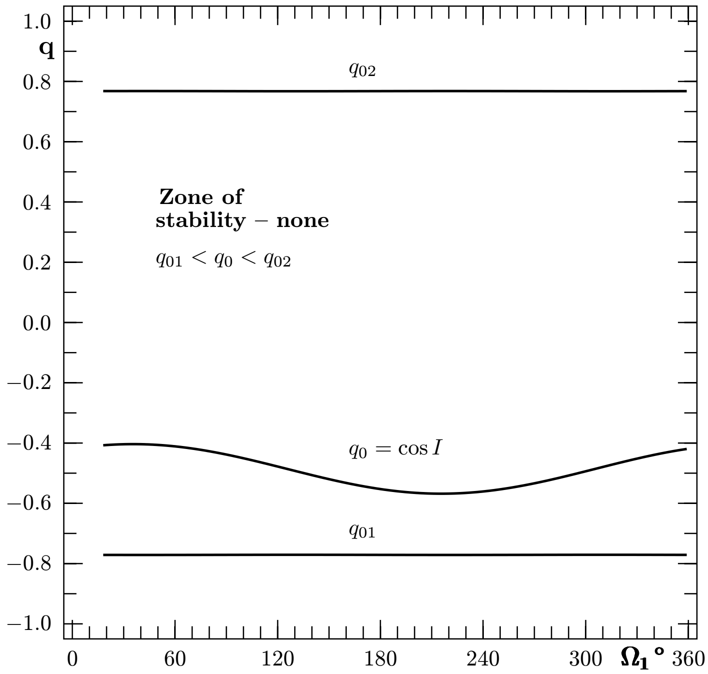

Figure 5. Theoretical boundaries of the stable zone and the cosine of the angle of the mutual inclination between planetary and the primary star orbits within the interval of the longitude of ascending node of the primary star $0^\circ < \Omega_1 < 360^\circ$.

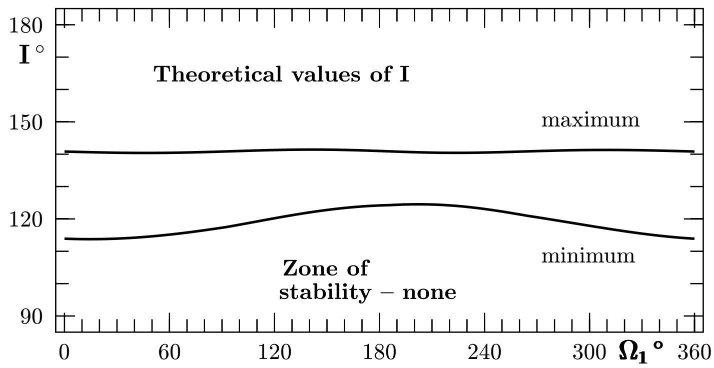

Figure 6. Theoretical minimum and maximum values of the mutual inclination I versus values of the longitude of ascending node of the primary star from the interval $0^\circ < \Omega_1 < 360^\circ$.

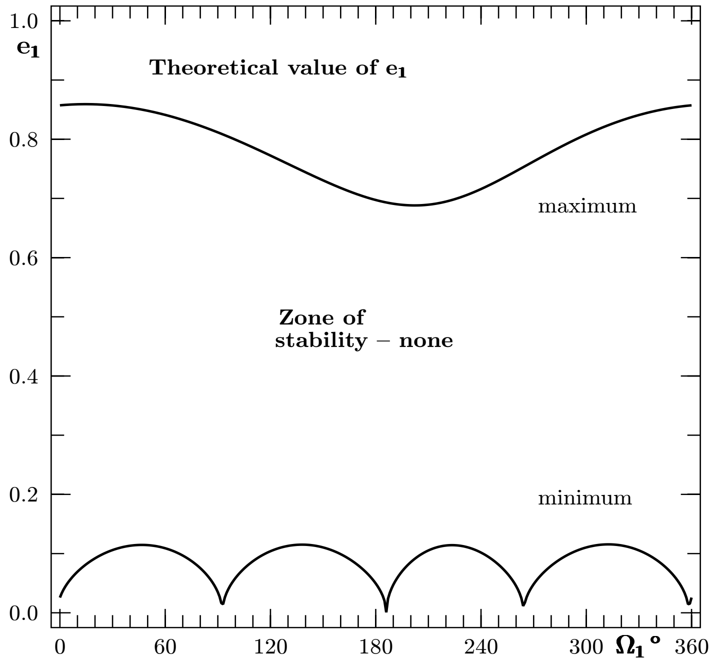

Figure 7. Theoretical minimum and maximum values of the eccentricity e_1 of the planet versus values of the longitude of ascending node of the primary star from the interval $0^\circ < \Omega_1 < 360^\circ$.

the interval (q_{01}, q_{02}). In Figure 6 theoretical minimum and maximum values of the mutual inclination I, in Figure 7 theoretical minimum and maximum values of the eccentricity e_1 of the planet are plotted. The results obtained from the theory are in good agreement with the results obtained by the numerical integration (see Figure 8).

For these initial data were calculated that the constant B is always negative. Therefore the system is unstable according to the Lyapunov conditions. It means that any small deviations from the initial elements of the planet may become substantial in its future motion. This can be also explained by a fact that the value of the cosine of the mutual inclination is close to the value of the cosine of the peculiar angle, which is defined by

$$q_p = -\frac{\eta}{2\overline{G}_2}, \tag{7}$$

where $\bar{c} = \overline{G}_2$ and $\eta = \sqrt{1 - e_1^2}$. It is a case when the orbit degenerates to a rectilinear segment.

If in the initial moment the angle of the mutual inclination of the orbits is close to the peculiar angle, then in the process of the dynamical evolution the

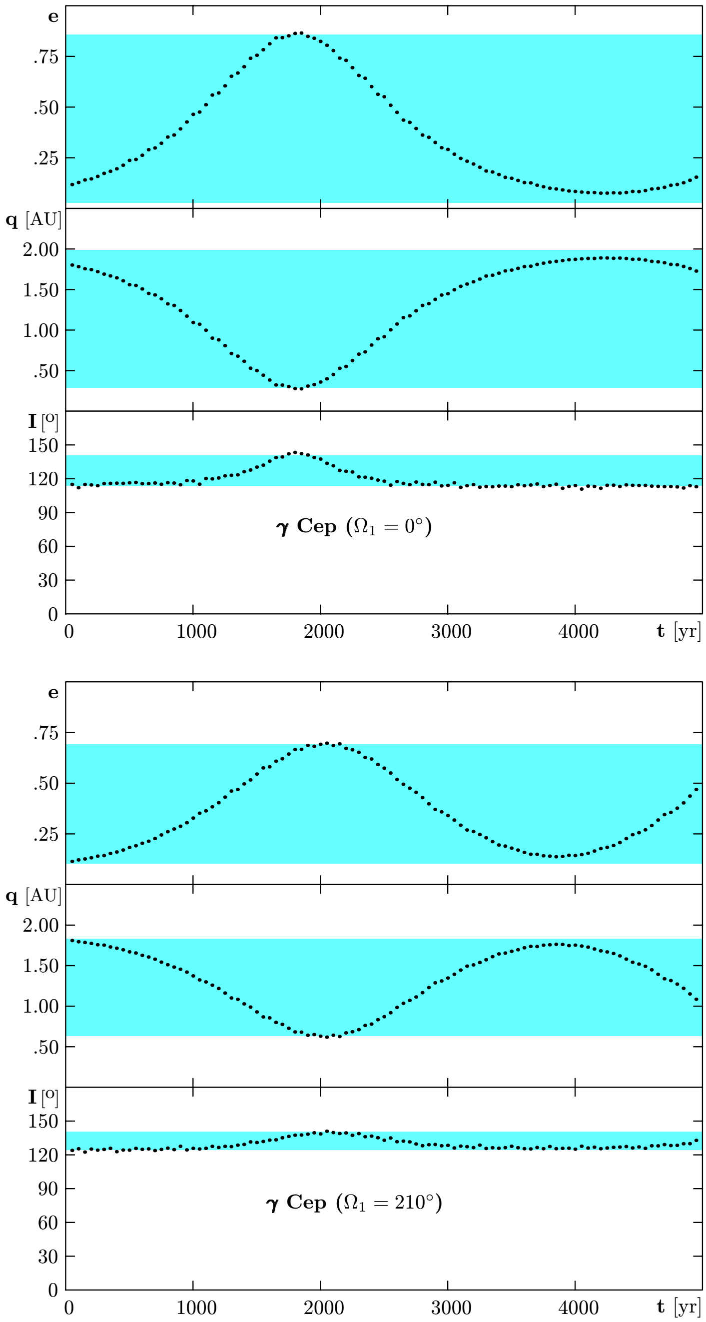

Figure 8. Orbital evolution of the planet of γ Cephei within the interval of 5000 years covering several revolutions of the distant star. Top for $\Omega_1 = 0^\circ$ and bottom for $\Omega_1 = 210^\circ$. Dots represent results obtained by the numerical integration. The gray zones are the boundaries of the orbital elements computed by the analytical theory.

system will reach the state, when the eccentricity e_1 of the planetary orbit will be close to 1, irrespective of the initial value of the eccentricity e_1. In our case the peculiar angle is $I_p = 90.5°$ and the angle of the mutual inclination of the orbits lie between 113° and 124°. For a given value of the primary star radius in the case of the largest value of the eccentricity, $e_1 = 0.859$, the planet in the perigee will move from the primary star's surface of $r_\pi = 0.266$ AU. Therefore the planet will be exposed to large tidal perturbations.

3. Conclusion

The results of our investigation indicate that the stability of an extra-solar planet orbit depends not only on the mass and distant ratios of the components, but also on the angle of mutual inclination between the planet and the distant star orbits, on the angular momentum of the system, and on the parameter $\overline{G}_2$. This parameter is a function of the ratio of the semi-major axes of orbits of the planet and the distant star, the eccentricity of the orbit of the distant star and masses of all components.

Our theory allows to predict the stability of the motion of an extra-solar planet over an astronomically long time interval. It allows to define ranges of unknown orbital elements within which the planetary orbit is stable, and to calculate the maximum and minimum values of the eccentricity of the planetary orbit.

The theory was demonstrated and verified by the numerical integration on the example of the γ Cephei system. The good agreement between the theoretical and numerical integration results was obtained.

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