Contrib. Astron. Obs. Skalnaté Pleso **38**, 12 – 20, (2008)

Radar meteors range distribution model

IV. Ionization coefficient

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Received: January 23, 2007; Accepted: September 25, 2007

Abstract. The theoretical radar meteors Range Distribution Model of the overdense echoes developed by Pecinová and Pecina (2007a) is applied here to observed range distributions of meteors belonging to the Quadrantid, Perseid, Leonid, Geminid, γ Draconid (Giacobinid), ζ Perseid and β Taurid streams to study the dependence of the ionization coefficient β on the meteoroid atmospheric velocity. The dependence in question is studied and confronted with the analogous dependence obtained by other authors. Also the mathematical form of a new dependence is evaluated and presented.

Key words: physics of meteors – radar meteors – range distribution – ionization coefficient

1. Introduction

We have developed the radar meteors range distribution model (RaDiM) that allows us to compute a few important parameters connected with meteor showers as well as with physical parameters of particles these showers consist of. Due to a huge amount of results we decided to publish them as a series of four articles. So, we divided the outcomes into four Papers sorted out logically in accord with its significance. Thus, while Paper I (Pecinová and Pecina, 2007 a) introduces and describes the method itself, Paper II (Pecinová and Pecina, 2007 b) deals with the flux density, Θ_{m_0}, and the mass distribution index, s, and Paper III (Pecinová and Pecina, 2007 c) concerns the ablation, shape-density and self-similarity (or Levin's) parameters. We focus here on the results concerning the ionization parameter (or sometimes also probability), β, we managed to achieve. We consider these results to be the most important of all. The ionization coefficient plays a very important role in physics of ionization of radar meteors. Further, it is known that β is a function of the atmospheric velocity of a meteoroid. This dependence has not been investigated properly so far. We succeeded in evaluation of β for seven meteor showers, i. e. for seven various velocities inherent to them. To sum up, we hope our results on this important parameter contribute to a better understanding of the meteor ionization process. We have also assumed the power velocity dependence of the β and have arrived at some numerical model of it. We give the corresponding formula at the end of this article where the comparison with results of other authors is made, too.

This article ends the whole series of works dealing with the radar meteors range distribution model. The method makes use of the rich observational data from the Ondřejov meteor radar which were collected during almost 50 years of observations.

2. Meteor ionization

Meteoric and atmospheric atoms and molecules become excited and ionized during inelastic collisions between the vaporizing atoms of a meteoroid and air atoms and molecules. As a result, an ion-electron trail forms along the path of a meteor. The trail as a whole is quasi-neutral. This fact enables the meteors to be studied also by means of radars transmitting electromagnetic waves which reflect on the trail and then can be studied since they bring useful information connected also with the properties of trail forming meteoroids and the structure of a meteor stream. The foundations of the theory of meteoric ionization were laid by Öpik (1933) and substantially improved by him in the fifties of the last century (Öpik, 1955, 1958).

The ionization coefficient β plays a very important role in the description of the meteor ionization process. Its definition claims that it equals the average number of free electrons created during the collisions of one evaporated meteor atom with other particles. It is included into the ionization equation linking the electron line density α_e to the mass loss of a meteoroid due to its ablation (e. g. Bronshten, 1983):

$$\alpha_e = -\frac{\beta}{\mu_a \, v} \left(\frac{dm}{dt} \right), \tag{1}$$

where the symbol μ_a stands for the average mass of an evaporated meteoroid atom. We usually adopt, after Ceplecha et al. (1998), the value $\mu_a = 40 \times \mu_H$ (where $\mu_H = 0.1673534056 \times 10^{-26}$kg is the mass of atomic hydrogen). The linear electron density α is directly proportional to β that decides about the ratio of energy consumed by ionization effects. This fact is very important. The quantity β is assumed to depend on meteor velocity in an unknown way. There have been a lot of attempts to describe this relationship between meteoroid's velocity and ionization probability. Several of them are listed in Table 23 in Bronshten (1983). Although it is possible to use any ionization theory we want in our model as the first estimate, we prefer here three of them, which seem to be the most plausible ones. Each of them is based on the assumption that the velocity dependence can be of the form of some power of meteoroid velocity. They are undermentioned below:

1. Verniani and Hawkins (1964) developed a relation based on observations with

$$\beta = \beta_c v^m, \tag{2}$$

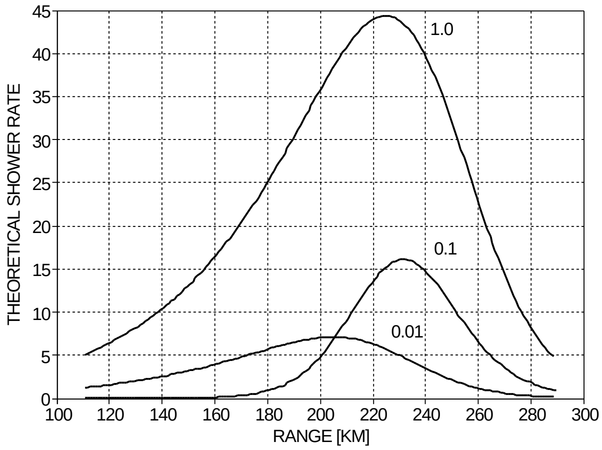

Figure 1. The course of the theoretical range distribution as a function of the ionization coefficient β. Each curve is marked by the corresponding value of β. The computations were performed for the Geminids between 1 and 2 UT, on the 13th of December, 2000. The constants and quantities used for the theoretical calculations were the following: mass $m_{\rm o} = 10^{-5}\,{\rm kg}$, $v_\infty = 36\,{\rm km\,s^{-1}}$, $K \cdot \sigma = 0.01\,{\rm s^2\,km^{-2}}$, $s = 1.5$, $\mu = 2/3$, $D_r = 4.2\,{\rm m^2\,s}$ (at the height of 93 km), $H = 5.409\,{\rm km}$ and $\varrho_o = 56.803\,{\rm kg\,m^{-3}}$. See also Paper I for the mathematics of the dependence.

where the constants $\beta_c = 0.1 \times 10^{-7}$ and $m = 4$.

2. Kashcheev et al. (1967) proposed a semiempirical model

$$\beta = \beta_k v^n, \tag{3}$$

with constants $\beta_k = 0.12649 \times 10^{-6}$ and $n = 3.5$ ($[v] =$ km s^{-1}).

3. Jones (1997) described the dependence in question in the following way:

$$\beta = \beta_o v^{n_1} (v - v_n)^{n_2}, \tag{4}$$

where $\beta_o = 0.94 \times 10^{-5}, n_1 = 0.8, n_2 = 2$ and $v_n = 10$ km s^{-1}. He developed a theoretical model valid only for velocities not exceeding 35 km s^{-1}.

Obviously, the course of a range distribution is modified by the value of the ionization coefficient β. The dependence is demonstrated in Fig. 1. On one hand, it can be seen that the lower the value of β the less pronounced the maximum is and wideness of the distribution bigger and vice versa. This fact corresponds with the ionization equation (1) and the picture of physical process within a meteor trail generally accepted. When keeping the atmospheric velocity of a meteoroid constant we can state that the bigger value of β the more massive the ionization (α_e) is and, consequently, also the bigger size of collecting area within

Table 1. Results of an application of the RaDiM to the Quadrantid stream meteors. The first column contains the year when the meteors were observed, the second one the day of observation, the third one the beginning hour of observation, bh, while the next the corresponding end hour, eh. The quantity $L_\odot$ is the solar longitude of the centre of an observation interval related to the equinox of J2000.0. The last column contains the values of the ionization coefficient β.

Year	Day	bh	eh	$L_\odot$	β		Year	Day	bh	eh	$L_\odot$	β
1961	3	0	2	$282\overset{\circ}{.}931$	0.100 ± 0.032	$\star$	1962	3	10	12	$283\overset{\circ}{.}093$	0.102 ± 0.043
1964	3	2	6	$282\overset{\circ}{.}270$	0.098 ± 0.033	$\star$	1965	3	10	12	$283\overset{\circ}{.}331$	0.105 ± 0.042
1966	3	8	10	$282\overset{\circ}{.}983$	0.111 ± 0.041	$\star$	1967	4	10	12	$283\overset{\circ}{.}821$	0.110 ± 0.039
1968	4	2	4	$283\overset{\circ}{.}227$	0.100 ± 0.042	$\star$	1969	3	4	5	$283\overset{\circ}{.}007$	0.110 ± 0.040
1975	4	4	6	$283\overset{\circ}{.}511$	0.100 ± 0.039	$\star$	1976	4	0	4	$283\overset{\circ}{.}130$	0.111 ± 0.043
1977	3	2	6	$282\overset{\circ}{.}953$	0.120 ± 0.052	$\star$	1978	3	2	4	$282\overset{\circ}{.}647$	0.098 ± 0.039
1980	4	1	5	$283\overset{\circ}{.}140$	0.108 ± 0.040	$\star$	1982	3	1	5	$282\overset{\circ}{.}626$	0.111 ± 0.041
1982	4	1	5	$283\overset{\circ}{.}646$	0.111 ± 0.039	$\star$	1983	4	3	5	$283\overset{\circ}{.}422$	0.100 ± 0.040
1985	3	9	15	$283\overset{\circ}{.}240$	0.107 ± 0.040	$\star$	1986	3	13	15	$283\overset{\circ}{.}060$	0.102 ± 0.020
1987	3	12	14	$282\overset{\circ}{.}757$	0.110 ± 0.040	$\star$	1987	4	4	6	$283\overset{\circ}{.}437$	0.105 ± 0.036
1988	4	3	5	$283\overset{\circ}{.}132$	0.110 ± 0.035	$\star$	1991	4	3	5	$283\overset{\circ}{.}366$	0.107 ± 0.036
1992	4	3	5	$283\overset{\circ}{.}110$	0.104 ± 0.042	$\star$	1992	4	1	5	$283\overset{\circ}{.}067$	0.110 ± 0.040
1994	3	1	5	$282\overset{\circ}{.}545$	0.099 ± 0.041	$\star$	1994	4	1	5	$283\overset{\circ}{.}564$	0.102 ± 0.040
1995	4	4	6	$283\overset{\circ}{.}390$	0.131 ± 0.045	$\star$	1995	4	10	12	$283\overset{\circ}{.}645$	0.111 ± 0.042
1996	4	3	5	$283\overset{\circ}{.}083$	0.120 ± 0.051	$\star$	1996	4	1	5	$283\overset{\circ}{.}041$	0.105 ± 0.040
1997	3	3	5	$282\overset{\circ}{.}820$	0.113 ± 0.042	$\star$	1997	3	1	5	$282\overset{\circ}{.}778$	0.108 ± 0.029
1998	3	2	6	$282\overset{\circ}{.}558$	0.111 ± 0.040	$\star$	1998	3	10	12	$282\overset{\circ}{.}855$	0.120 ± 0.040
1999	4	2	6	$283\overset{\circ}{.}311$	0.115 ± 0.040	$\star$	1999	4	2	4	$283\overset{\circ}{.}269$	0.101 ± 0.035
2000	4	2	4	$283\overset{\circ}{.}012$	0.109 ± 0.044	$\star$	2000	4	10	12	$283\overset{\circ}{.}351$	0.105 ± 0.035
2001	3	0	4	$282\overset{\circ}{.}710$	0.102 ± 0.044	$\star$	2001	3	10	14	$283\overset{\circ}{.}135$	0.110 ± 0.045
2001	3	10	12	$283\overset{\circ}{.}092$	0.103 ± 0.043	$\star$	2002	3	12	14	$282\overset{\circ}{.}913$	0.090 ± 0.035
2004	4	1	5	$282\overset{\circ}{.}986$	0.111 ± 0.044	$\star$	2005	3	10	12	$282\overset{\circ}{.}568$	0.107 ± 0.036
2005	3	10	14	$283\overset{\circ}{.}109$	0.111 ± 0.041	$\star$						

the echo plane. Moreover, this is in agreement with relation (38) from Paper I. The magnitude of the maximum of an ionization curve is directly proportional to the value of β but the height at which this maximum occurs does not depend on it. On the other hand, the whole situation is complicated by the fact that β is velocity dependent. The effect of atmospheric velocity on the range distribution curve is discussed in Paper I in more detail.

3. Results and discussion

We applied RaDiM to 7 meteor showers observed by the radar at Ondřejov observatory. These are: the Quadrantids 1961-2005, the Perseids 1980-2000, the Leonids 1965-2002, the Geminids 1959-2001, the γ Draconids (Giacobinids) 1998 and the daytime showers, ζ Perseids and β Taurids 2003. In total, we make use of 127 observed range distributions to get the following 5 parameters: the shower

Table 2. The same as in Table 1 but for Perseids.

Year	Day	bh	eh	$L_\odot$	β
1980	12	10	10	$140\overset{\circ}{.}111$	0.216 ± 0.078
1981	11	22	4	$139\overset{\circ}{.}463$	0.197 ± 0.061
1981	12	0	2	$139\overset{\circ}{.}463$	0.212 ± 0.083
1982	12	22	24	$140\overset{\circ}{.}100$	0.184 ± 0.069
1982	12	22	4	$140\overset{\circ}{.}180$	0.208 ± 0.087
1983	12	22	24	$139\overset{\circ}{.}856$	0.202 ± 0.072
1983	13	0	2	$139\overset{\circ}{.}936$	0.209 ± 0.080
1985	13	2	4	$140\overset{\circ}{.}483$	0.202 ± 0.073
1985	13	12	14	$140\overset{\circ}{.}883$	0.208 ± 0.070
1986	13	0	2	$140\overset{\circ}{.}640$	0.212 ± 0.082
1989	12	8	12	$139\overset{\circ}{.}780$	0.199 ± 0.089
1991	13	0	2	$139\overset{\circ}{.}890$	0.192 ± 0.077
1992	11	22	2	$139\overset{\circ}{.}599$	0.212 ± 0.072
1993	12	12	16	$139\overset{\circ}{.}913$	0.210 ± 0.082
1995	14	4	10	$141\overset{\circ}{.}311$	0.212 ± 0.074
1996	12	0	6	$139\overset{\circ}{.}695$	0.199 ± 0.068
2000	12	6	10	$139\overset{\circ}{.}873$	0.202 ± 0.077
2000	12	6	10	$139\overset{\circ}{.}873$	0.212 ± 0.069

flux density Θ_{m_0}, the mass distribution index s, the self-similarity (or Levin's) parameter μ, the product of $K \cdot \sigma$ and ionization parameter β. The results on the first 4 parameters have been described in Papers II and III. This paper deals with the last one. The results in question we have managed to achieve for particular meteor showers are collected in Tables 1 - 6.

From Table 1 we can see the limits inside which the β ranges in the case of the Quadrantid meteor stream. In order to be able to characterize the meteors of a particular shower as to the magnitude of β, and since we assume that all meteors of a shower move with almost the same velocity, we should have a quantity at hand which would be common to all shower meteors. This can be, similarly to analogous quantities from Paper I or II, the weighted mean of particular β values included in Table 1. The result will be presented together with other values of β later on. We can state that all values of β we have found for Quadrantids are greater than those given by Verniani and Hawkins (1964) and Kashcheev et al. (1967). The comparison with Jones (1997) cannot be performed because of the velocity limit of his formula.

The corresponding result of the application of RaDiM to Perseids is listed in Table 2. We have obtained the values that are between the Verniani and Hawkins (1964) and Kashcheev at al. (1967) curves. Again, no comparison with Jones (1997) can be made. The weighted mean was again calculated and is presented later on.

Table 3. The same as in Table 1 but for Leonids.

Year	Day	bh	eh	$L_\odot$	β
1965	17	4	8	$235\overset{\circ}{.}123$	0.346 ± 0.100
1966	17	0	4	$234\overset{\circ}{.}700$	0.347 ± 0.121
1966	17	4	8	$234\overset{\circ}{.}868$	0.322 ± 0.110
1998	17	0	2	$234\overset{\circ}{.}448$	0.324 ± 0.113
1998	17	3	4	$234\overset{\circ}{.}531$	0.349 ± 0.100
1998	17	7	8	$234\overset{\circ}{.}699$	0.332 ± 0.123
1999	18	4	6	$235\overset{\circ}{.}369$	0.338 ± 0.099
2000	18	1	3	$235\overset{\circ}{.}988$	0.332 ± 0.104
2001	18	12	13	$236\overset{\circ}{.}155$	0.359 ± 0.118
2001	19	1	4	$236\overset{\circ}{.}786$	0.352 ± 0.127
2002	19	1.5	4.5	$236\overset{\circ}{.}526$	0.342 ± 0.110

Table 4. The same as in Table 1 but for Giacobinids observed on October 8, 1998.

Year	Day	bh	eh	$L_\odot$	β
1998	8	12	14	$195\overset{\circ}{.}028$	0.029 ± 0.006

The similar process as in the case of previous two streams can also be applied to Leonids and the results of application are presented in Table 3. Also in the case of Leonids our resulting values of β are mostly between Verniani and Hawkins (1964) and Kashcheev at al. (1967) curves. The weighted mean of β will also be presented later on.

A good example of a shower with low velocity but high ablation ability is the γ Draconid (Giacobinid) one. The result on β valid for this shower is included in Table 4. We can see that our result is higher than that following from Jones (1997) formula. We cannot calculate any weighted mean here so that the value we have arrived at is used directly in establishing the velocity dependence of β.

The other stream we made use of is the Taurid meteoroid complex, namely its daytime showers ζ Perseids and β Taurids. The result we obtained is in Table 5. We can state in the connection with our result that the values of β are

Table 5. The same as in Table 1 but for ζ Perseids observed on June 8, 2003, and β Taurids observed on June 25, 2003.

Year	Day	bh	eh	$L_\odot$	β
			ζ Perseids		
2003	8	4	8	$76\overset{\circ}{.}982$	0.071 ± 0.007
			β Taurids		
2003	25	5	8	$93\overset{\circ}{.}233$	0.047 ± 0.006

Table 6. The same as in Table 1 bur for the Geminid meteor shower.

Year	Day	bh	eh	$L_\odot$	β		Year	Day	bh	eh	$L_\odot$	β
1959	13	2	6	260°.916	0.074 ± 0.029	$\star$	1960	13	2	6	261°.667	0.087 ± 0.032
1961	14	0	4	262°.342	0.080 ± 0.032	$\star$	1962	12	0	4	260°.044	0.085 ± 0.028
1963	12	20	24	260°.629	0.078 ± 0.022	$\star$	1964	11	20	24	260°.375	0.082 ± 0.036
1965	12	20	24	261°.126	0.083 ± 0.030	$\star$	1965	13	20	24	262°.142	0.078 ± 0.023
1966	14	0	2	262°.061	0.081 ± 0.031	$\star$	1967	13	0	4	260°.776	0.082 ± 0.029
1968	13	0	4	261°.531	0.075 ± 0.030	$\star$	1969	12	0	4	260°.259	0.084 ± 0.028
1969	12	4	8	260°.429	0.077 ± 0.030	$\star$	1969	14	0	4	262°.292	0.079 ± 0.029
1973	12	0	4	260°.225	0.082 ± 0.027	$\star$	1974	14	0	4	261°.998	0.079 ± 0.030
1975	13	0	4	260°.725	0.082 ± 0.029	$\star$	1975	14	0	4	261°.741	0.076 ± 0.023
1976	13	0	4	261°.477	0.082 ± 0.028	$\star$	1977	12	0	4	260°.204	0.083 ± 0.030
1977	13	0	4	261°.221	0.088 ± 0.036	$\star$	1978	12	2	4	259°.984	0.081 ± 0.027
1978	14	2	4	262°.017	0.082 ± 0.028	$\star$	1980	12	2	4	260°.484	0.080 ± 0.028
1980	13	2	4	261°.501	0.084 ± 0.028	$\star$	1981	10	4	6	258°.273	0.078 ± 0.022
1981	12	2	4	260°.222	0.082 ± 0.035	$\star$	1981	14	2	4	262°.253	0.087 ± 0.031
1982	13	0	6	260°.849	0.082 ± 0.025	$\star$	1982	14	0	6	261°.991	0.083 ± 0.027
1984	10	4	6	258°.500	0.080 ± 0.022	$\star$	1985	13	0	4	261°.165	0.079 ± 0.021
1986	13	0	4	260°.903	0.081 ± 0.028	$\star$	1986	14	0	2	261°.878	0.086 ± 0.031
1987	15	0	4	262°.674	0.083 ± 0.031	$\star$	1989	13	0	4	261°.139	0.085 ± 0.031
1989	14	0	4	262°.155	0.081 ± 0.031	$\star$	1990	13	0	4	260°.876	0.080 ± 0.027
1991	14	0	4	261°.638	0.085 ± 0.027	$\star$	1992	12	0	4	260°.358	0.084 ± 0.030
1994	12	1	5	259°.883	0.083 ± 0.032	$\star$	1995	13	0	4	260°.589	0.083 ± 0.033
1995	14	0	4	261°.606	0.080 ± 0.026	$\star$	1996	12	2	4	260°.374	0.082 ± 0.034
1997	13	0	4	261°.085	0.081 ± 0.026	$\star$	2000	12	0	4	260°.303	0.084 ± 0.026
2000	13	0	4	261°.320	0.088 ± 0.033	$\star$	2000	14	0	4	262°.336	0.080 ± 0.029
2000	13	1	5	261°.362	0.088 ± 0.035	$\star$	2001	13	1	5	261°.102	0.083 ± 0.030

quite close to those following from Jones (1997) formula.

The last stream we have employed in our investigation is the Geminid one. The relevant results are in Table 6. Its velocity of 36 km s^{-1} is quite close to the limit of Jones (1997). We can check from Table 6 that our resultant values of β are lower than the value computed using the Jones formula (4). Also for this shower the weighted mean has been determined and is presented later on.

We have assumed that shower meteoroids we used for our analysis are not decelerated in the atmosphere. As a consequence, β in our computations was only a number inherent to a particular shower. This fact enables us to establish the velocity dependence of β when employing showers of different velocities directly from the observations without need of any theory of β. Thus, we arrived at ionization coefficients for 7 different values of meteor velocities. We also assumed the ionization probability β to be the same for all members of the same meteor shower and computed weighted mean values of it. The result that is summarized in Table 7 is also depicted in Fig. 2 where the curves following from the Verniani and Hawkins (1964) formula together with the one of Kashcheev

Table 7. Weighted means of the ionization coefficient β valid for the particular stream. For the corresponding preatmospheric velocity of the particular shower, see e. g. Paper I. It is expressed in km/s.

Shower	β	v_∞
Quadrantids	0.107 ± 0.010	43
Perseids	0.205 ± 0.001	61
Leonids	0.343 ± 0.001	71
Geminids	0.082 ± 0.001	36
Giacobinids	0.029 ± 0.010	23
β Taurids	0.071 ± 0.007	32
ζ Perseids	0.047 ± 0.006	29

et al. (1967) and Jones (1997) are drawn, too. We can readily see that at low velocities only ζ Perseids and β Taurids conform to the Jones formula. Our β value of Giacobinids is greater than that of Jones while the corresponding ionization probabilities of Geminids and Quadrantids are smaller. All showers with $v_\infty \leq 43\,\mathrm{km\,s^{-1}}$ have provided us with β greater than that of Kashcheev et al. (1967). The values of Perseids and Leonids lie between the ones of Verniani and Hawkins (1964) and Kashcheev et al. (1967).

We have decided to fit our results on β according to the values from Table 7 by means of the least-square method assuming velocity dependence of the form $\beta(v) = \beta_p\, v^p$ with the following result:

$$\beta_p = (4.66 \pm 1.84) \times 10^{-5}, \qquad p = 2.1 \pm 0.1\,.$$

4. Conclusions

We have applied the radar meteors range distribution model developed by Pecinová and Pecina (2007 a) to 7 showers in order to get the values of the ionization coefficient velocity dependence. The result is listed in Table 7 and is also drawn in Fig. 2. Comparison with the results of other authors has revealed that for velocities $v_\infty \geq 35\,\mathrm{km\,s^{-1}}$ our function gives the values of β between those of Verniani and Hawkins (1964) and Kashcheev et al. (1967) with the exception of Quadrantids and Geminids for which our value exceeds the values given by both groups. Our results on ζ Perseids and β Taurids are quite close to the values predicted by the work of Jones (1997) while for γ Draconids (Giacobinids) our results exceeds that of Jones (1997).

We would like to stress here that we have not used any theory of β and our results are, therefore, of an observational character. We hope our results will contribute to other development of the theory of the ionization coefficient. Unfortunately, RaDiM is a method that does not allow us to study the ionization coefficient in a more detailed way. Its velocity dependence is a very fine effect from the point of this method.

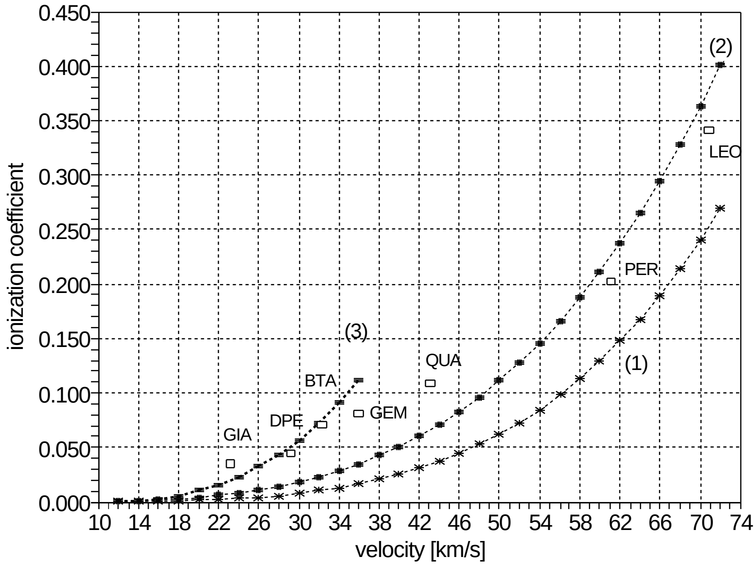

Figure 2. The ionization probability β as a function of velocity v_∞ of the particular shower. The curves (1), (2) and (3) are drawn according to Verniani and Hawkins (1964), Kashcheev et al. (1967) and Jones (1997) respectively. The empty rectangles that are described by the name of a particular shower represent our results (see Table 7).

Acknowledgements. This work has been supported by the Project AV0Z10030501 and partly by the grant No. 205/03/1405 of the Grant Agency of the Czech Republic.

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