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The impact orbits of the dangerous asteroid (99942) Apophis

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Abstract. The impact solutions of the dangerous asteroid (99942) Apophis were obtained from computations performed with the use of the OrbFit software. For all the computations the OrbFit Software, Package 3.3.2, was used. Precise impact orbits for all impact solutions of Apophis predicted for the 2007 epoch and three impact orbits for epochs close to the impact dates in 2036, 2037 and 2054 were computed. The computations of impact orbits were made according to Sitarski (2006) using the OrbFit package and the interpolation method. Moreover, we found out additional dates of impacts of Apophis, especially in 2038.

Key words: asteroids – celestial mechanics

1. Introduction

In the paper, we investigated the motion of Apophis - the most dangerous asteroid for the Earth according to the Impact Risk Page of NASA: *http://neo.jpl.nasa.gov/risk/* with the greatest (June 2007) Palermo Scale value equal -2.5. A similar value was computed by the NEODyS. The Palermo Technical Impact Hazard Scale was developed to classify the impact risk including not only impact dates, but also the energies and probabilities of the potential impact.

To compute the exact impact solutions of asteroids, it is necessary to include some additional small effects influencing the asteroid's motion. In the previous work (Wlodarczyk, 2007b) we investigated the influence of relativistic effects and perturbing massive asteroids with the use of Solex90 of Vitagliano (1997) software and different ephemeris of the Solar System. With the new version of OrbFit Package 3.3.2 (http://newton.dm.unipi.it/neodys/astinfo/orbfit/), we can compute impact probability mainly due to the use of non-linear monitoring and a multiple-solution method (Milani *et al.*, 2002, Milani, 2005a and Milani *et al.*, 2005b).

The main goal of our work was to compute the impact orbits using the OrbFit software combined with the method depicted by Sitarski (2006) and employing interpolation of neighboring clones of the impact clone.

Given complicated propagation of errors of computing orbital elements of dangerous asteroids, such as Apophis, Wlodarczyk (2007a) shows that the determination of impact orbits is a difficult problem.

2. The starting orbital elements for (99942) Apophis and the numerical testing

2.1. The starting orbital elements of Apophis

Orbital elements given in Table 1 were computed by the author using all 1007 observations up to now (there have been no new observations since 16/8/2006) and the OrbFit software where M denotes mean anomaly, a - semimajor axis, e - eccentricity, ω_{2000} - argument of perihelion, Ω_{2000} - longitude of the ascending node, i_{2000} - inclination of the orbit, and *nom.* is the nominal orbit. These orbital elements are referred to the *J2000* equator and equinox.

Table 1. Starting orbital elements of (99942) Apophis. Angular elements ω, Ω, i refer to Equinox J2000.0 (Epoch: 20070410=JD2454200.5).

orbit	Orbital elements		
nom.	M=307 °3630792962062	a= 0.922261414460134	e= 0.191059417047684
	ω =126°3855624046634	Ω =204°4591613325997	i=3°3313147101503
	arc: 2004–2006	number of obs.: 1007	rms =0.″302

The computed orbital elements are almost the same as those listed on the NASA JPL site, for example, the semimajor axis differs by 7.0 10^{-10} AU (i.e. about 100 m). The computed semimajor axis has an uncertainty (1-σ variation) of about 2.4 10^{-8} AU (3600 m).

The orbital elements of Apophis given in Table 1 were computed using the JPL Planetary and Lunar Ephemerides DE406, including relativistic effects. To compute the impact solutions for Apophis precisely, it is necessary to include gravitational perturbations of massive asteroids. We have taken into account perturbations from 4 massive asteroids (Ceres, Pallas, Vesta and Hygiea) with masses taken from the JPL NASA. The small differences between this orbit of Apophis and that presented in the previous paper (Wlodarczyk, 2007b) are connected with the use of different JPL Planetary and Lunar Ephemerides and different masses of perturbing asteroids.

2.2. Numerical testing

To estimate the errors of the computation of the orbital elements of Apophis the forward and then backward integration of the equations of motion was carried out as a standard procedure. The computations were made using the package OrbFit. The equations of motion of the nominal orbit of Apophis from Table 1

were integrated from the epoch of start 2007/04/10 (MJD54200) to 2023/02/25 (MJD60000) and results were stored in a file every 100 days. Then the file with orbital elements for a given epoch MJD was saved and became the input orbital elements file. Next, we computed residuals (*rms*) and the covariance matrix (no correction). The difference between the starting orbital elements and those from backward integration gave the backward integration to the starting epoch.

Similar computations were made for integration from starting orbital elements to 2050/07/13 (MJD70000) and to 2083/05/21 (MJD82000) and backward.

The results are given in Table 2 where M is the mean anomaly, a - semimajor axis, e - eccentricity, ω_{2000} - argument of perihelion, Ω_{2000} - longitude of the ascending node, i_{2000} - inclination of the orbit, and Δ - differences in values of orbital elements between the starting orbit and that obtained by backward integration. This Table shows that the errors (Δ) of the integrations of the equations of motion are quite small. Only in one orbital element, namely in the mean anomaly, the difference between the starting value and the one obtained after forward and backward integration reaches about $0^\circ.00003$ in all presented tests. The errors of the other computed orbital elements are much smaller. The *rms* of the computed orbits in years 2023 and 2050 are quite small, below $3''.0$. Hence the computed results of integration of the equations of motion of Apophis can be applied in this work for the presented time spans.

Table 2. Apophis - Differences between the forward and backward integration of the nominal orbit of Apophis. (Epoch: 20070410=JD2454200.5).

	$M/\Delta M$	$a[AU]/\Delta a$	$e/\Delta e$
start	$307^\circ.3630792962062$	0.922261414460134	0.191059417047684
2023 backward	$0^\circ.0000286$	0.0000000009	0.00000000081
2050 backward	$0^\circ.0000287$	0.0000000009	0.00000000086
2083 backward	$0^\circ.0000286$	0.0000000009	0.00000000081
	$\omega_{2000}/\Delta\omega$	$\Omega_{2000}/\Delta\Omega$	$i_{2000}/\Delta i$
start	$126^\circ.3855624046634$	$204^\circ.4591613325997$	$3^\circ.3313147101503$
2023 backward	$0^\circ.000000868$	$0^\circ.000000217$	$0^\circ.000000095$
2050 backward	$0^\circ.000000867$	$0^\circ.000000217$	$0^\circ.000000096$
2083 backward	$0^\circ.000000868$	$0^\circ.000000217$	$0^\circ.000000095$

3. Close approaches to planets

Close approaches to planets are very sensitive to the perturbing model of the Solar System with included massive asteroids. In Wlodarczyk (2007b) the influ-

ence of the approaching asteroids on the motion of Apophis was presented. Using Vitagliano's package SOLEX90 we found the asteroids with close approach to Apophis. Now, we compare three main perturbing models of asteroids included in the SENTRY, the NEODyS and the one proposed by the author, as well as a model without perturbing asteroids.

Table 3 lists the close approaches of the nominal orbit to within 0.1 AU of the terrestrial planets. *CPVHE* denotes used perturbations from Ceres, Pallas, Vesta, Hygiea and Eros; *CPVH* - as above but without Eros, *CPV* - perturbations from Ceres, Pallas and Vesta and (–) denotes perturbing model without asteroids. In all cases relativistic effects were included.

We can see in Table 3 that the differences between close approaches solutions became visible after a very close approach in 2029 when Apophis passes the Earth at a distance of about 38,000 km. It is clear that the use of perturbing asteroids has a great influence on the motion of Apophis and, hence, on the impact solutions. Especially, the influence of Eros is well visible as it was stated in our previous work (Wlodarczyk, 2007b).

A similar analysis of influence of asteroids on the motion of asteroid 1950DA was performed by Giorgini *et al.* (2002), where 61 asteroids - perturbers changed the along-track position of 1950DA by -1.2 days after 879 years.

Table 3. Apophis - Close approaches to the terrestrial planets from 2004 to 2088 within 0.1 AU.

Planet	Date	MJD	Nominal distance [AU]	Perturbing asteroids
EARTH	2004/12/21.39225	53360.39225	0.09638388	CPVHE
	2004/12/21.39225	53360.39225	0.09638387	CPVH
	2004/12/21.39225	53360.39225	0.09638387	CPV
	2004/12/21.39225	53360.39225	0.09638387	–
EARTH	2013/01/09.48827	56301.48827	0.09666137	CPVHE
	2013/01/09.48829	56301.48829	0.09666139	CPVH
	2013/01/09.48830	56301.48830	0.09666139	CPV
	2013/01/09.48828	56301.48828	0.09666140	–
VENUS	2016/04/24.11792	57502.11792	0.07824136	CPVHE
	2016/04/24.11792	57502.11792	0.07824131	CPVH
	2016/04/24.11792	57502.11792	0.07824131	CPV
	2016/04/24.11792	57502.11792	0.07824130	–
EARTH	2021/03/06.05183	59279.05183	0.11265128	CPVHE
	2021/03/06.05188	59279.05189	0.11265123	CPVH
	2021/03/06.05188	59279.05188	0.11265123	CPV
	2021/03/06.05188	59279.05188	0.11265123	–
EARTH	2029/04/13.90710	62239.90710	0.00025517	CPVH
	2029/04/13.90710	62239.90710	0.00025499	CPVH
	2029/04/13.90710	62239.90710	0.00025499	CPV
	2029/04/13.90710	62239.90710	0.00025498	–

Table 3. Continued.

Planet	Date	MJD	Nominal distance [AU]	Perturbing asteroids
EARTH	2037/09/22.20556	65323.20556	0.17250000	CPVHE
	2037/09/22.55505	65323.55505	0.18067944	CPVH
	2037/09/22.55416	65323.55416	0.18065865	CPV
	2037/09/22.55878	65323.55878	0.18076729	–
EARTH	2044/07/18.10030	67814.10030	0.12100386	CPVHE
	2044/10/04.61615	67892.61615	0.17256625	CPVHE
	2044/07/23.08891	67819.08891	0.11744526	CPVH
	2044/07/23.07581	67819.07581	0.11745551	CPV
	2044/07/23.14420	67819.14420	0.11740197	–
EARTH	2051/04/09.96629	70270.96629	0.05567206	CPVHE
	2051/04/11.07097	70272.07097	0.03895038	CPVH
	2051/04/11.06803	70272.06803	0.03899279	CPV
	2051/04/11.08333	70272.08333	0.03877154	–
EARTH	2059/09/15.39268	73351.39268	0.03707626	CPVHE
	2059/09/13.62691	73349.62691	0.02224563	CPVH
	2059/09/13.63581	73349.63581	0.02226721	CPV
	2059/09/13.58894	73349.58894	0.02216037	–
EARTH	2066/03/22.52528	75731.52528	0.17598468	CPVHE
	2066/06/08.70421	75809.70421	0.11947570	CPVHE
	2066/05/01.56002	75771.56002	0.07356518	CPVH
	2066/05/02.03855	75772.03855	0.07441701	CPV
	2066/04/29.56167	75769.56167	0.06980108	–
EARTH	2073/04/05.43581	78302.43581	0.13905693	CPVHE
EARTH	2081/09/16.87220	81388.87220	0.06243259	CPVHE
	2081/08/19.51205	81360.51205	0.08856791	CPVH
	2081/08/20.27526	81361.27526	0.08752678	CPV
	2081/09/16.48064	81357.48064	0.09259025	–
EARTH	2088/07/04.99026	83871.99026	0.12618025	CPVHE
	2088/04/13.14253	83789.14253	0.00720571	CPVH
	2088/04/13.23862	83789.23862	0.00615500	CPV
	2088/04/12.75919	83788.75919	0.01146441	–

4. Impact solutions of Apophis

After the orbital elements for Apophis were determined, the impact Table was computed. Table 4 lists impact solutions for (99942) Apophis computed by the author for the following settings: multiple solutions, using scaling, line of variation (LOV) with the largest eigenvalue, 6 σ, 1200 steps on each side of the nominal orbit (Milani *et al.* 2002). Hence, we have computed 2401 multiple solutions, i.e. 2401 orbits (clones) are available. Next we conducted a close ap-

proach analysis for the whole selected interval between 1-st and 2401-st clone. Finally, we searched for close approaches until the given year and clones (virtual asteroids (VA)) were propagated and close approaches as well as impact dates were detected. The OrbFit software ver. 3.3.2 for UNIX was used. In this impact Table weighing of observations was the same as in CLOMON2. Notes a and b in Table 4 denote two possible different impact solutions for the same year.

Table 4. Impact Table for (99942) Apophis

date	MJD	σ	dist. *[RE]*	impact probab.	note
2036/04/13.371	64796.371	-2.46675	1.15	2.09E-05	
2037/04/13.644	65161.644	4.26592	1.36	4.51E-08	
2038/04/13.658	65526.658	4.27099	1.58	3.73E-11	
2042/04/13.715	66987.715	-2.49073	2.08	5.72E-08	
2044/04/13.295	67718.295	-2.43012	2.08	3.60E-08	a
2044/04/13.262	67718.262	4.23311	1.96	1.96E-10	b
2053/04/12.913	71004.913	-2.44040	1.40	1.09E-07	
2054/04/13.402	71370.402	4.36807	1.88	1.45E-09	
2056/04/13.194	72101.194	4.29935	1.36	8.45E-11	
2059/04/13.953	73196.953	4.21507	2.07	1.07E-10	
2063/04/13.792	74657.792	4.36400	1.32	4.35E-11	a
2063/04/13.800	74657.800	4.36400	1.66	3.21E-11	b
2068/04/12.631	76483.631	-0.10008	0.58	1.80E-06	
2069/04/13.078	76849.078	2.11580	0.97	4.42E-07	a
2069/10/15.970	77034.970	1.46985	0.28	5.57E-07	b
2078/04/13.445	80136.445	2.11508	1.44	8.37E-09	

In Table 4 *date* is a calendar day for the potential impact; *MJD* - Modified Julian Day ($JD = 2400000.5 + MJD$); σ - the location along the LOV in σ space; values of σ are in the interval [-6,6] which represent 99.999999 % probability of occurrence of a real asteroid in this confidence region; *distance [RE]* - minimum distance, the lateral distance from LOV (line of variation which represents the central axis of the asteroid's elongated uncertainty region); *impact probab.* - impact probability computed with the Gaussian bidimensional probability density (the impact probability is not reported if the computed value is less than $1.0 \cdot 10^{-11}$).

5. Impact orbits of Apophis

The problem of computing impact orbits appeared while studying the motion of asteroid 1999 AN10 and 1997XF11 (Sitarski, 1999). In his work Sitarski found

impact orbits by the Cracovian least squares correction using the forced equality constraints. He also computed two impact orbits for hypothetical collisions of 1997 XF11 with the Earth in 2028 and 2033, and four orbits for 1999 AN10 colliding with the Earth in 2027, 2034, 2036, and 2039. If we find an impact orbit of the dangerous asteroid, we can predict the impact place on the Earth. Sitarski (2006) presented the method of generating clones of an impact orbit for four chosen asteroids: 2004VD17, 1950DA, Apophis and Hathor.

Now we propose the method of computing the impact orbits of Apophis using the OrbFit software and an interpolation method.

Firstly, to get precise impact orbits we generated 10000 virtual asteroids as well as the nominal orbit of Apophis. We used the same observations of Apophis as in the case of computing the impact Table (Table 4) in the previous section. The computed value of σ were taken from impact solutions for a 6 σ interval. We increased the number of clones (virtual asteroids) from 2400 to 10000 to decrease the distance between the clones on LOV. First, we computed the serial number of the impact clone (NC). In the case of negative σ:

$$NC = \frac{6+\sigma}{6} \cdot 5000 + 1, \tag{1}$$

and for positive σ:

$$NC = \frac{\sigma}{6} \cdot 5000 + 5001, \tag{2}$$

The computed serial numbers of impact clones (NC) for different impact solutions are listed in Table 5. For each impact date there are: rounded σ of the impact solution from Table 4; *rms* of the nominal orbit from a multiple solution, in our case 5000 clones exist on each side on LOV and, hence, the nominal orbit has the number 5001; the serial number of the impact clone computed from equation (1) or (2), NC and serial numbers of neighboring clones with the impact clone with their *rms*. In all computations the exact values of σ from Table 4 were used. For example, for the impact in 2036 the NC=2945.37, i.e. the impact orbit is on LOV between clones with serial number 2945 (clone a) and *rms=0.32"* and 2946 (clone b) with *rms=0.31"*. The computations were made for one or two epochs close to the impact date, mostly about seven days before the impact. This time span was proposed by Sitarski (2006).

Precisely, to compute orbital elements of the impact orbit, Lagrange's interpolation formula of 12-th order was used. At the beginning the impact orbits of Apophis for the epoch of starting orbital elements (2007/04/10) were computed. This date was proposed by Sitarski (private information). It tourned out that differences between orbital elements computed from a different serial number of neighboring clones are negligible. Therefore, we used 6 neighboring clones on each side of the impact orbit. The computed impact orbits for the epoch of starting orbital elements (2007/04/10) are presented in Table 6.

As we can see in Table 5, the *rms* of the nominal orbit of Apophis ($\sigma = 0$, serial number 5001 in all 10000 clones with the central nominal orbit) is below

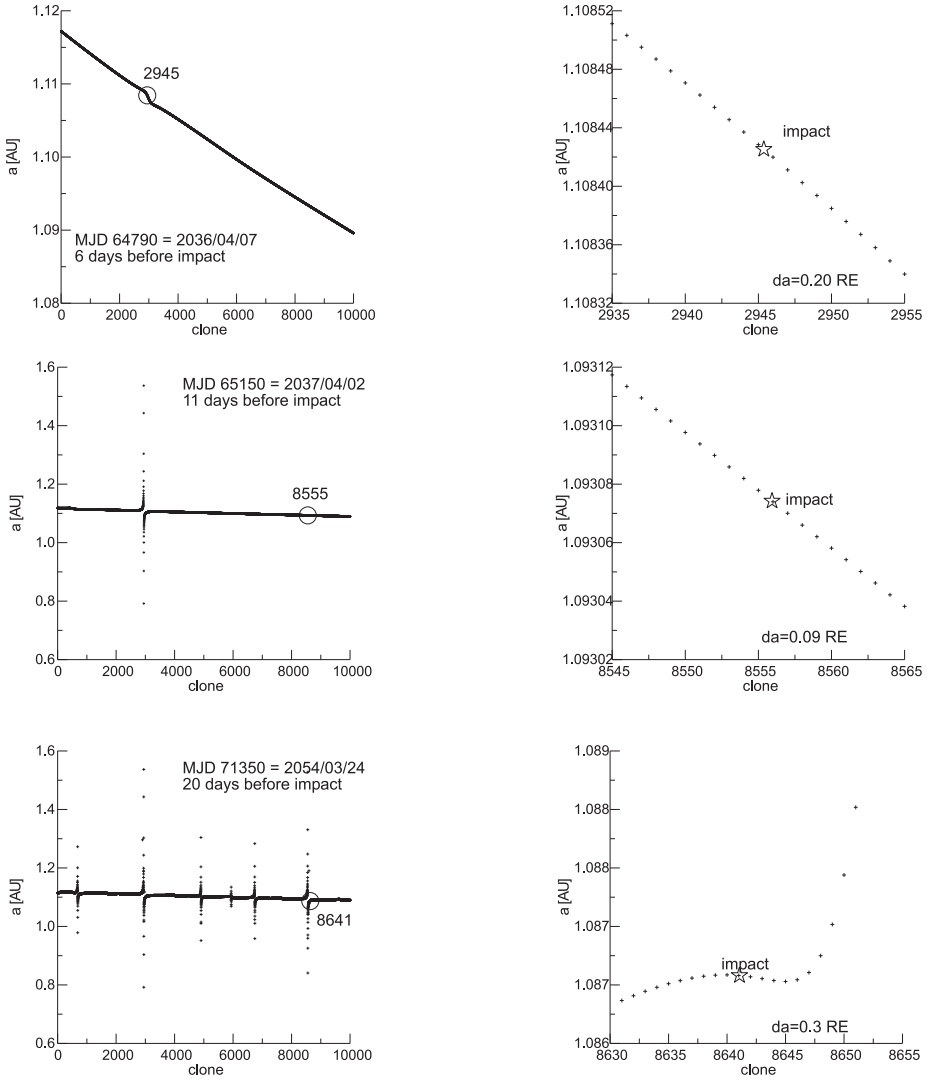

Figure 1. The propagation of semimajor axes for different clones of Apophis for epochs close to the impact solutions - left panels, and on the right - the neighboring clones of the impact clone with the computed value *da*, the difference between neighboring clones in Earth radii, *RE*. Impact years 2036, 2037 and 2054.

1" till the year 2056 except for the epoch MJD 71000 in the year 2053. But it is not so good for clones a and b, which are further from the nominal orbit and have $\sigma \neq 0$. Only for clones in 2036 and 2037 the *rms* are smaller than 1". Hence, the computed impact orbits of Apophis in Table 6 for the epoch 2007/04/10 are precise despite the additional errors connected with the interpolation method. The computed *rms* of the impact orbits of Apophis are almost the same as for its starting orbital elements.

Table 5. Impact clones and their neighboring clones of (99942) Apophis for the epochs close to the impact dates

impact		epoch	σ	nom	impact clone	clone a	clone b
date	MJD	MJD		rms	NC	clone /rms	clone /rms
2036/04/13.371	64796.371	64790	-2.467	0.31	2945.37	2945/0.32	2946/0.31
2037/04/13.644	65161.644	65150	4.266	0.32	8555.94	8555/0.34	8556/0.37
2038/04/13.658	65526.658	65510	4.271	0.36	8560.16	8560/204	8561/114
		65520		0.31		8560/107	8561/46.9
2042/04/13.715	66987.715	66970	-2.491	0.50	2925.39	2925/113	2926/39.4
		66980		0.30		2925/33.3	2926/0.41
2044/04/13.295	67718.295	67700	-2.430	0.43	2975.90	2975/9.2	2976/15.5
		67710		0.65		2975/15.1	2976/30.6
2044/04/13.262	67718.262	67700	4.233	0.43	8528.59	8528/11.9	8529/6.8
		67710		0.65		8528/41.1	8529/18.1
2053/04/12.913	71004.913	70990	-2.440	0.65	2967.33	2967/141	2968/2.2
		71000		1.24		2967/84.4	2968/125
2054/04/13.402	71370.402	71350	4.368	0.42	8641.06	8641/2.53	8642/1.52
2056/04/13.194	72101.194	72081	4.300	0.48	8583.80	8583/111	8584/1.06
2059/04/13.953	73196.953	73180	4.215	1.55	8513.56	8513/90.9	8514/53.8
2063/04/13.792	74657.792	74640	4.364	7.72	8637.66	8637/34.3	8638/74.0
2063/04/13.800	74657.800	74640	4.364	7.72	8637.66	8637/34.3	8638/74.0
2068/04/12.631	76483.631	76460	-0.100	11.8	4917.60	4917/192	4918/40.5
2069/04/13.078	76849.078	76830	2.116	7.44	6764.17	6764/11.3	6765/475
2069/10/15.970	77034.970	77015	1.470	22.6	6225.87	6225/72.6	6226/64.2
2078/04/13.445	80136.445	80120	2.115	41.9	6763.57	6763/101	6764/4903

But on approaching the epochs of orbital elements of clones closer to the impact dates we encounter difficulties with the interpolation method and with *rms* of clones. The propagation of the semimajor axes of the clones of Apophis are presented in Fig. 1. On the left panels the semimajor axes of all 10000 clones of Apophis together with the nominal orbit are presented. As early as in 2037 we observed rapid growth of distances between several of clones. Hence, it is difficult

Table 6. The impact orbits of (99942) Apophis for the epoch 20070410 = JD2454200.5, J2000. Angular elements ω, Ω, i refer to Equinox J2000.0.

imp. year /rms	Orbital elements		
2036	M=307°3631513056707	a=.9222613560438911	e=.1910595639474889
0.307	ω=126°3855553251768	Ω=204°4591704378594	i=3°331313810832751
2037	M= 307°362954765407	a=.9222615154832834	e=.1910591630041577
0.317	ω=126°3855746476322	Ω=204°459145586326	i=3°331316265394649
2054	M=307°3629517835907	a=.9222615179023332	e=.1910591569204789
0.318	ω=126°3855749406676	Ω=204°4591452093245	i=3°331316302640217

Table 7. Impact orbital elements of (99942) Apophis for epochs several days before the date of the impact. Angular elements ω, Ω, i refer to Equinox J2000.0.

imp. year	Orbital elements		
2036	M=303°9189196298411	a=1.108425261852979	e=.1913969402630119
	ω=70°31090322022615	Ω=203°4803540987443	i=2°168275679022818
	Impact:2036/04/13.371		Epoch:2036/04/07
2037	M=295°7114277158994	a=1.093074219159016	e= .1865296949441665
	ω=73°93925962521709	Ω=203°4928305672	i=2°323677326982186
	Impact:2037/04/13.644		Epoch:2037/04/02
2054	M=286°486555164992	a= 1.086580125193066	e=.1847302259271072
	ω=75°34632325632232	Ω=203°0524369782738	i=2°344865350437748
	Impact:2054/04/13.402		Epoch:2054/03/24

Table 8. Impact solutions from Mercury package

date of impact	distance to the Earth	
	AU	RE
2036/04/13.37	0.000015	0.35
2037/04/13.64	0.000020	0.47
2054/04/13.40	0.000042	0.99

to compute the exact value of orbital elements by the interpolating method. However, the distances between clones are not equally spaced. In addition, as it is computed in Table 5, *rms* of the neighboring clones has a value much greater than *rms* of the starting orbital elements.

On the right panels the distances, *da* between neighboring clones in Earth radii, *RE* are shown. For "easy" (regular) cases as for the year 2036 and 2037, this distance is smaller than one Earth radius. A similar value is obtained for the year 2054.

It is of interest that in Fig. 1 the rapid jumps in semimajor axes appear with the time which complicate the computations of impact orbits using the presented interpolation method. But even in the "regular" evolution of semimajor axes of clones on left panels or right panels the computed impact orbits using the interpolation method are influenced by high *rms* about several minutes or degrees of arc. Therefore, we reduced computing of the impact orbits for 2036, 2037 and 2054 years only.

Next, the impact orbits were computed for all the epochs close to the impact dates, several days before the impact. The epochs were the same as in Fig. 1.

These impact orbits presented in Table 7 can be easily used to determine the impact point location on the Earth's surface according to Sitarski's (2006) suggestion, or to compute the impact solution from a different existing software. For example, we can use the popular software Mercury Integrator Package v. 6.0 of J. Chambers (1999). The results of integration of impact orbits from Table 7 using the Mercury software are presented in Table 8, where only those impact solutions which give the minimum distance to the Earth are considered. These results coincide with the results given in Table 4.

6. Summary

We have computed the exact impact orbits of Apophis for all dates of impact solutions for the epoch 2007 and three impact orbits of Apophis for the epoch close to the impact dates in 2036, 2037 and 2054. However, our results show that it is possible to compute impact orbits near the impact dates using the OrbFit package. We hope to compute new exact impact solutions with the use of higher value of σ and a greater number of clones on LOV. Then it will be possible to compute impact orbits close to the impact dates using spaced clones of greater density near the impact clone (NC) and by applying the interpolation method.

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