

Observational tests of general relativity and alternative theories of gravity with Galactic Center observations using current and future large observational facilities

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Abstract. It is established that there are supermassive black holes in centers of galaxies. A supermassive black hole with mass around $4 \times 10^6 M_{\odot}$ is located at the Galactic Center. Such an approach for the Galactic Center looks rather natural, in spite of that consequences of model must be checked with observations. We discuss opportunities to check this with forthcoming observations of shadows in mm band for the Galactic Center as it was done recently for M87*. Observations of bright stars moving near the Galactic Center gives another opportunity to evaluate gravitational potential. We discuss opportunities to use these observations to constrain alternative theories of gravity.

Key words: supermassive black holes – general relativity – alternative theories of gravity

1. Introduction

Astronomers believe that supermassive black holes are located in centers of galaxies, including our Galaxy. However, for the Galactic Center theorists proposed many different models (including exotic ones), such as a dense cluster of stars (Reid, 2009), fermion balls (Munyanzeza & Viollier, 2002), boson stars (Jetzer, 1992; Torres, Capozziello & Lambiase, 2000), neutrino balls (De Paolis et al., 2001). Later, some of these models have been ruled out, or the range of parameters of these models is significantly constrained with consequent observations (Reid, 2009).

Now it is accepted that there is a super-massive black hole at the Galactic Center (see, e.g., recent reviews Reid 2009; Zakharov 2017, 2018a, 2019).

It is well-known that the universal gravity law was discovered with efforts of J. Kepler, R. Hooke and I. Newton, see, for instance, a popular interesting review on the subject by Arnold (2000). Using planet observations Kepler found three laws of planet motions in our Solar system, Hooke understood that Kepler's laws could be explained if gravity force is inversely proportional to squared distance and wrote the idea in his letter to Newton (Arnold, 2000; Taton & Wilson, 2003). Newton proved that the Hooke'd hypothesis is correct and vice versa Newton showed that the inverse square law follows from Kepler's law on ellipticity of stellar orbits. Therefore, a law for an interaction could be obtained from an analysis of trajectories of moving objects. Similarly, later E. Rutherford used α particles to investigate an atomic structure and A. Einstein created a modification of Newtonian gravity and explained the Mercury anomaly with his theoretical approach.

To evaluate its gravitational potential near a black hole, one can analyze trajectories of test particles moving in the potential, and as a result one could constrain the parameters of the potential. In this article we discuss opportunities to use photons or massive objects as test particles to evaluate gravitational potential near the Galactic Center using the conventional approach of general relativity and models obtained in the framework of alternative theories of gravity. If we speak about observable quantities at the first case we consider size and shape of shadows around supermassive black holes in Sgr A* and M87*, at the second case, one analyzes trajectories of bright stars near the Galactic Center.

2. Shadows around black holes

As it was noted one could use photons as test particles and analyze the images around supermassive black holes including the Galactic Center. We remind papers where the authors discussed issues connected with the subject. Assuming that there is a luminous screen behind a black hole Bardeen considered the apparent shape of a Kerr black hole located between a luminous screen and a distant observer (Bardeen, 1973). Critical sets of impact parameters occupied by plunge parabolic orbits of particles and of photons has been considered by Young (1976) for the Kerr – Newman – de-Sitter metric (the apparent shape of a black hole could be obtained from the region of critical photon orbits, with a reflection in respect to the rotation axis). Later, Luminet (1979) showed a silhouette of a Schwarzschild black hole, and images of thin accretion disks around a spherical symmetric black hole. However, earlier, visible shapes of circular orbits around Schwarzschild and Kerr black holes had been shown by Cunningham & Bardeen (1973), see also beautiful pictures of accretion disks around a Kerr black hole had been reproduced recently for the *Interstellar* movie (James et

al., 2015), where one could recognize shadows since for rotating black holes the authors considered truncated disks with inner radii $r_{\text{inner}} > r_{\text{ISCO}}$.

It is well-known that photon geodesics in a Kerr metric are characterized by only two parameters (integrals of motion), called ξ and η (Chandrasekhar, 1983). One could introduce a function $\eta_{cr}(\xi)$ of critical values of ξ and η which correspond to unstable spherical photon orbits with $r = \text{const}$ (r is the radial Boyer – Lindquist coordinate). This set separates plunge and scatter regions (Zakharov, 1986). In the book by Chandrasekhar (1983), a parametric representation of the functions $\eta_{cr}(r)$ and $\xi(r)$ was used. Such a representation of these functions used by Chandrasekhar (1983) is not suitable since a topology of sets $\{\xi, \eta\}$ corresponding to scatter and plunge regions is not presented properly, for instance, in Fig. 39 in book by Chandrasekhar (1983) curve $\eta_{cr}(r)$ is shown for $\eta < 0$, while it is known that for pairs $(\xi, \eta) \in M = \{(\xi, \eta) | -a^2 + 2a|\xi| - \xi^2 \leq \eta < 0\}$ one has photon capture since geodesics with constants of motion correspond to vortex motion and these photon geodesics do not cross the equatorial plane (Yakovlev, 1975), while for other pairs (ξ, η) with $\eta < 0$ photon geodesics do not exist. In addition, in paper by Zakharov (1986) it was shown that maximum $\eta_{cr}(r)$ is 27 and it corresponds to $\xi = -2a$. This useful property of $\eta_{cr}(r)$ function was not discussed by Chandrasekhar (1983). Such a property could be expressed as a property of shadow for observers in the equatorial plane as it was done by Zakharov et al. (2005a) and now it is widely used in the literature to evaluate a spin from future shadow observations.

We would like to remind that in paper by Zakharov (1986), it was noted that this function $\eta_{cr}(\xi)$ separates scatter and plunge photon orbits, namely, photons are plunging for positive η only if $(\xi, \eta) \in S$, where

$$S = \{(\xi, \eta) | 0 \leq \eta < \eta_{cr}(\xi) \quad \& \quad \xi_1 < \xi < \xi_2\}, \quad (1)$$

and ξ_1 and ξ_2 are the critical impact parameters of the retrograde and direct unstable photon circular equatorial orbits, respectively (Zakharov, 1986); all length quantities are expressed in M units, while η is expressed in M^2 units

$$\xi_1 = -6 \cos \left(\frac{\arccos a}{3} + \frac{2}{3}\pi \right) - a \quad (2)$$

and

$$\xi_2 = 6 \cos \frac{\arccos(-a)}{3} - a. \quad (3)$$

Pairs $(\xi, \eta) \in S$ correspond to double roots of the polynomial $R(r)$ governing the radial motion of photons (Zakharov, 1986) (we would like to note that this condition is valid for black holes but not for naked singularities). In addition, in paper by Zakharov (1986) it was proven that the maximal value of the function $\eta_{cr}(\xi)$ is 27 and $\eta_{cr}(-2a) = 27$; the radial Boyer – Lindquist coordinate value for this orbit reads $r(-2a) = 3$. Therefore, one can see that the representation of the function $\eta_{cr}(\xi)$, as it was done by Zakharov (1986), is more clear than

the parametric representation of this function as it was done by Chandrasekhar (1983), and one could easily recognize scatter and plunge orbits, and prove that the maximal value of $\eta_{cr}(\xi)$ does not depend on the black hole spin a .

At the first glance, a model with a luminous screen behind a black hole, studied by Bardeen (1973); Chandrasekhar (1983), does not look realistic, because in astronomy there is no luminous screen behind a black hole, and the sizes of a silhouette (shadow) are too small to be detectable in seventies and eighties of the last century for masses and distances of known black holes – for example the super-massive black hole at the Galactic Center has the angular size of the shadow, as observed from the Earth, around $50 \mu as$, however, now this shadow size is large enough to be reconstructed with advanced VLBI facilities in the mm-band (similarly to the Event Horizon Telescope) or more precisely bright structures with such sizes could be observed and dark shadow could be reconstructed from observed distribution of bright structure. In this case, one could say that it would be possible to observe unseeable (Doeleman, 2017).

Based on ideas introduced by Chandrasekhar (1983), Zakharov (1986) and Holz & Wheeler (2002), Zakharov et al. (2005a) reformulated results about the properties of the $\eta_{cr}(\xi)$ function obtained earlier by Zakharov (1986), and considered different types of the shadow shapes for the Kerr black holes, and different position angles of a distant observer. In addition, Zakharov et al. (2005a) showed that for an equatorial plane position of a distant observer, maximal impact parameter $|\beta_{max}|$ in the z -direction (which coincides with the black hole rotation-axis direction) is $\sqrt{27}$ (in GM/c^2 units), and $\beta_{max} = \sqrt{27}$ for $\alpha = 2a$, or $|\beta(2a)| = \sqrt{27}$ (Zakharov et al., 2005a), if we consider the function $\beta(\alpha)$ for the critical impact parameters separating plunge and scatter regions of photons ($\beta(\alpha)$ is expressed through function $\eta_{cr}(\xi)$ and a position angle of a distant observer). It means that for an observer in the equatorial plane, $|\beta_{max}|$ remains the same, the shadow is deformed in the direction which is parallel to equatorial plane, and such a deformation depends on the black hole spin a . This theoretical property of the black hole shadow is widely used to evaluate the black hole spin from observations. For instance, Hioki and Maeda (2009) proposed to use such parameters (radius and distortion parameter for shadows) to evaluate the black hole spin from observations. Therefore, evaluations of the shadow sizes around the black holes could help to estimate of the black hole parameters (Zakharov et al., 2005a,b; Zakharov, 2014, 2015; Cherepashchuk, 2016, 2017; Bisnovatyikogan & Tsupko, 2017; Dokuchaev & Nazarova, 2017; Cunha & Herdeiro, 2018; Shaikh, 2018, 2019; Dokuchaev & Nazarova, 2019). Opportunities to compare predictions of general relativity and alternative theories of gravity with measuring the shadow size for the black hole at the Galactic Center has been discussed by Zakharov et al. (2012); Johannsen et al. (2016).

Some time ago Falcke, Melia & Agol (2000); Melia & Falcke (2001) simulated the shadows for supermassive black holes and showed that the black hole silhouette could be formed in a rather natural way (see also Falcke, Melia & Agol (2013); Johannsen (2016) for more recent reviews). The authors used a toy

model for their analysis, and they concluded that the strong gravitational field bends trajectories of photons emitted by accreting particles, and an observer could see a dark spot (shadow) around a black hole position or more precisely one could reconstruct a shape of shadow analyzing bright structure distributions. For the black hole at the Galactic Center the size of the shadow is around $3\sqrt{3}R_S$ (where $R_S \approx 10 \mu\text{as}$), as is the angular size of the Schwarzschild radius. Based on results of simulations, Falcke, Melia & Agol (2000); Melia & Falcke (2001) concluded that the shadow may be detectable at mm and sub-mm wavelengths, however, scattering may be very significant at cm wavelengths, so there are very small chances to observe the shadows at the cm band. The ground – space interferometer Radioastron (which was launched in 2011 and it finished its operation in the beginning of 2019) had the shortest wave length around 1.3 cm, therefore, it was no chance to reconstruct shadow structure in spite of nice angular resolution around $7 \mu\text{as}$ at 1.3 cm. We should mention that the results obtained by Falcke, Melia & Agol (2000); Melia & Falcke (2001) are rather general, in spite of their specific model. Strictly speaking, it is impossible to see darkness (shadows) in astronomical observations and people try to investigate structures of bright spots near shadows since shadows are formed by envelopes of bright images – analyzing structures of images one could reconstruct shadows (Broderick & Loeb, 2006). Further simulations and observations for M87* were confirmed these claims.

There is a tremendous progress in evaluation of minimal size of a spot detectable by recent observational techniques near the Sgr A^* (Shen et al., 2005; Doeleman et al., 2008; Doeleman, 2008). For example, Doeleman et al. (2008); Doeleman (2008) evaluated a bright spot size as small as $37_{-10}^{+16} \mu\text{as}$ for the VLBI technique in mm-band, but a boundary of a dark spot (shadow) has to be bright, and the related size of the bright boundary has been evaluated, and, therefore, the theoretical estimate of the shadow size and the bright spot size obtained from the observations should have similar values. These activities, including design and construction of new facilities, observations, and data analysis, are important steps to create the so-called the Event Horizon Telescope (Doeleman et al., 2009; Doeleman, 2017), see also for a more recent information¹. The idea is to create a world-wide VLBI network to observe pictures of the supermassive black hole at the Galactic Center and in the galaxy M87 center. As the project authors claimed, they are developing a Earth size telescope because lengths of arms are comparable with the Earth diameter, however, one should remind that earlier the authors of the ground – space interferometer Radioastron declared that they created the telescope much bigger than Earth since its longest arm length is about $3 \times 10^5 \text{ km}$ (Kardashev, 1997, 2009, 2013). As it was mentioned earlier, initially, it was expected to analyze bright accretion structures near the black hole horizon at the Galactic Center, but consequent observations and estimates showed that the shortest wavelength of Radioastron is around 1.3 cm,

¹<http://eventhorizontelescope.org/>.

and it is too long to observe a shadow at the Galactic Center since electron scattering is blurring a shadow image. A turbulence is an important issue and it could distort images of the bright spots near the shadows (Broderick et al., 2016).

2.1. Shadow reconstruction for M87*

On April 10, 2019 the Event Horizon Telescope (EHT) collaboration reported results of the shadow reconstruction for observations of M87* on four days in April 2017 at 1.3 mm wavelength (EHT Collaboration: Akiyama et al., 2019a,b,c,d,e,f). M87 is a giant elliptical galaxy with a rather massive black hole in its center. Earlier, there were different estimates of the black hole mass at M87*. Data analysis of EHT observations in April 2017 supported a black hole mass estimate around $6.5 \times 10^9 M_{\odot}$, which was also discussed earlier in the literature. Distance toward M87 is around 17 Mpc. The shadow diameter is around $(42 \pm 0.3) \mu as$.² Accuracy of image reconstruction is not better than $25 \mu as$, it means that the image reconstruction presented by EHT Collaboration: Akiyama et al. (2019a) is not unique. Generally speaking, these results are consistent with predictions in the framework of conventional black hole model in general relativity, therefore, general relativity passed one significant test more. However, many alternatives for this approach are not rule out yet (EHT Collaboration: Akiyama et al., 2019a), it is rather natural since a number of alternatives (sometimes exotic ones) have very similar features observed with EHT as it was demonstrated earlier in computer simulations, differences for alternative approaches may be very tiny, as shown by (Vincent et al., 2016; Mizuno et al., 2018) where the authors discussed shadow formation for the boson star and the black hole models, see also discussion by Cunha et al. (2015, 2016).

3. Observations of bright stars near the Galactic Center as tool to evaluate gravitational potential

The closest supermassive black hole is located in our Galactic Center and astronomers observe the Galactic Center in different spectral bands. Moreover, such an object is a natural laboratory to test general relativity and check its possible alternatives in a weak gravitational field limit and in the future in a strong gravitational field limit. In 2018 the GRAVITY collaboration reported about the discovery of general relativity effects for S2 star observations near its pericenter passage in May 2018 (GRAVITY Collaboration: Abuter et al., 2018a). We would like to remind that British astronomers observed light deflection from foreground stars during Solar eclipse in May 1919 (Dyson, Eddington

²In spite of different distances toward Sgr A* and M87* and different black hole masses in these objects angular sizes of Schwarzschild radii (therefore shadow diameters) have similar sizes. This case reminds the coincidence of angular sizes for Sun and Moon.

& Davidson, 1920) and they tested three different options, namely, a) there is no a deflection of light by gravitating body; b) a deflection of light is describing by a Newtonian theory; c) a deflection of light is describing by general relativity and after an analysis of their observations astronomers concluded that general relativity is better fitting observations and we had to adopt general relativity instead of Newtonian theory as an universal gravity theory. Similarly, the GRAVITY team showed that general relativity (or more precisely the first post-Newtonian correction of general relativity) is much better describing relativistic redshifts for S2 star near its pericenter passage. It means that general relativity passed one significant test more and we have an additional argument that general relativity is an universal theory of gravity.

There is a very interesting and challenging opportunity to monitor bright IR stars moving around the Galactic Center. Two groups of astronomers use such an opportunity and observe these stars with largest telescopes which are equipped with adaptive optics facilities. American group led by A. Ghez uses the twin Keck 10 m telescopes and this group participates in a development of the Thirty Meter Telescope at Hawaii. Another European group uses four VLT telescopes which have four 8 m telescopes and four 1.8 m telescopes and these telescopes joined in the GRAVITY interferometer facilities. Later, European astronomers will use the European Extremely Large Telescope (E-ELT) to improve an accuracy of bright star orbit reconstruction. Observations of bright stars showed that these stars move along elliptical orbits and therefore, one could conclude that a gravitational potential of point like mass around $M_{SBH} = 4 \times 10^6 M_{\odot}$ could be adopted as the first approximation. After than one could try to find deviations from elliptical orbits which could represent the relativistic effect which is similar to Mercury anomaly discovered by U. Le Verrier in 1859 and explained by A. Einstein in 1915. S2 star is one of the most interesting object to test a gravitational potential at the Galactic Center. This star has eccentricity $e = 0.88$, period $T = 16$ yr and an expected visible relativistic precession of its orbit is around $\Delta s \approx 0.83$ mas (Gillessen et al., 2017; Chu et al., 2017) if we assume that extended mass distributions inside its orbit do not have a significant impact on relativistic precession. Currently the Keck uncertainty in the S2 star orbit reconstruction is around $\sigma_{Keck} \approx 0.16$ mas (Hees et al., 2017), while for Thirty Meter Telescope(TMT) which will be constructed within a several years $\sigma_{TMT} \approx 0.015$ mas.

4. First discoveries with GRAVITY

4.1. Gravitational redshift of S2 star near its pericenter passage

There is a rapid improvement of accuracy of S2 star orbit reconstruction, for example for the European team, since in 1990s a precision of SHARP facilities were around 4 mas, in 2000s NACO had a precision around 0.5 mas, but in 2018 GRAVITY reached a precision around $30 \mu as$ (GRAVITY Collaboration:

Abuter et al., 2018a). It is well-known that measurement of gravitational redshift is one of the classical tests of general relativity. The GRAVITY collaboration estimated gravitational redshift in the orbit of S2 star near its pericenter passage in May 2018 and showed that observational data are much better fitted with GR model in the first PN approximation in comparison with Newtonian one. Therefore, general relativity successfully passed its test for the Galactic Center. It means that almost after 100 years since the confirmation of the GR prediction about a deflection of light during Solar eclipse in 1919 Dyson, Eddington & Davidson (1920), astronomers checked GR prediction at high distances from our Solar system. A theoretical background for gravitational redshift evaluation if sources are moving in binary system was developed by Kopeikin & Ozernoy (1999); Alexander (2005); Zuker et al. (2006). S2 star passed pericenter in May 2018 and after analysis of these observational data it was clear that relativistic corrections for gravitational redshift have to be taken into account near this passage. At the pericenter S2 moves with a total space velocity $V_{\text{peri}} \approx 7650$ km/s therefore $\beta_{\text{peri}} = V_{\text{peri}}/c = 2.55 \times 10^{-2}$ (GRAVITY Collaboration: Abuter et al., 2018a). Considering the first post-Newtonian correction for a total gravitational redshift could expressed in the following form (Kopeikin & Ozernoy, 1999; Alexander, 2005; Zuker et al., 2006; GRAVITY Collaboration: Abuter et al., 2018a)³

$$z_{\text{GR}} = \frac{\Delta\lambda}{\lambda} = B_0 + B_{0.5}\beta + B_1\beta^2 + \mathcal{O}(\beta^3), \quad (4)$$

where $B_1 = B_{1,tD} + B_{1,grav}$, $B_{1,tD} = B_{1,grav} = 0.5$, $B_{1,tD}$ is the special relativistic transverse Doppler effect, $B_{1,grav}$ is the general relativistic gravitational redshift, $B_{0.5} = \cos\theta$, where θ is the angle between the velocity vector and line of sight (Alexander, 2005), the redshift B_0 is independent on a star velocity β and

$$B_0 = z_{\odot} + z_{\text{gal}} + z_{\text{star}} + \frac{1}{2}\Upsilon_0, \quad (5)$$

therefore, the redshift B_0 consists of four terms, the first term z_{\odot} is due a total motion of the Sun and the Earth in respect to Galactic Center and blue shift due to potential of the Sun and the Earth, z_{gal} is redshift due to Galaxy potential, z_{star} is redshift due to the star's potential, the redshift $\frac{1}{2}\Upsilon_0 = \frac{GM}{2a}$ due to the location of star in the SMBH potential (Alexander, 2005). The GRAVITY collaboration evaluated the total redshift from spectroscopical observations $z \approx \frac{200 \text{ km/s}}{c}$ (GRAVITY Collaboration: Abuter et al., 2018a). One could represent the total redshift obtained from spectroscopical observations in the following form (GRAVITY Collaboration: Abuter et al., 2018a)

$$z_{\text{tot}} = z_K + f(z_{\text{GR}} - z_K), \quad (6)$$

³Results of precise measurements of gravitational redshift with Galileo satellites on elliptical orbits have been published recently (Delva et al., 2018; Herrmann et al., 2018).

where $z_K = B_0 + B_{0.5}\beta$ is the Keplerian redshift, $f = 0$ corresponds to Keplerian (Newtonian) fit, while $f = 1$ corresponds to PPN(1) fit. The GRAVITY collaboration found that $f = 0.90 \pm 0.09|_{\text{stat}} \pm 0.15|_{\text{sys}}$ and the authors also concluded that S2 data are inconsistent with a pure Newtonian dynamics with a very high confidence level. Later, the estimate for redshift parameter has been corrected $f = 1.04 \pm 0.05$ with a significance level around 20σ (GRAVITY Collaboration: Abuter et al., 2019). Similarly, the GRAVITY collaboration evaluated f -value from observational data comparing Schwarzschild precession and Newtonian fit for a point like mass (without any precession) and they concluded that the f -value is much closer to GR quantity ($f = 0.94 \pm 0.09$). The Keck team obtained similar results (Do et al., 2019), namely, the authors found that $f = 0.87 \pm 0.17$ and the Newtonian model $f = 0$ has to be rejected with 5σ confidence level.

A comprehensive review on constraints of alternative theory parameters from observations of bright stars around the Galactic Center is given by GRAVITY Collaboration: Amorim et al. (2019).

4.2. Observations of motions of hot spots near the innermost stable circular orbit (ISCO)

The GRAVITY collaboration reported about observations of two bright flares near the Galactic Center on July 22 and July 28, 2018, as well as a fainter flare on May 27, 2018 (GRAVITY Collaboration: Abuter et al., 2018b). The authors noted that the position centroids exhibited clockwise looped motion on the sky, on scales of typically $150 \mu\text{as}$ over a few tens of minutes, corresponding to about 30% the speed of light. Meanwhile, the flares exhibited continuous rotation of the polarization angle, with about the same $45(\pm 15)$ min period as that of the centroid motions. These observations are very important to reconstruct magnetic field distribution near the Galactic Center. Typical radius of spot orbits are around $7 M_{\text{SBH}}$ (in mass units), while the ISCO radius is $6 M_{\text{SBH}}$ for a Schwarzschild black hole. It means the motions of these flares is very close to boundary of stability for bounded orbits.

4.3. Spatially resolved rotation of broad line region for 3C273

The GRAVITY collaboration observed not only the Galactic Center but also bright quasars including 3C273. Recently the authors reported observations of a spatial offset (with a spatial resolution of 10^{-5} arcseconds, or about 0.03 parsecs for a distance of 550 million parsecs) between the red and blue photo-centres of the broad Paschen- α line of the quasar 3C 273⁴ perpendicular to the direction

⁴3C 273 is the first quasar ever been identified. Initially it was observed with Parkes Radio Telescope by Lunar occultation method. M. Schmidt found its redshift $z \approx 0.158$ with Palomar 200-in telescope (Schmidt, 1963). 3C273 is the closest and brightest quasar. Analyzing old photo plates where there were 3 C273 images, Soviet astronomers and Sharov and Efremov found variabilities with time scale T_{var} around a few days (Sharov & Efremov, 1963). It means that a size 3 C273 is less than cT_{var} (where c is a speed of light) or less than the Solar system

of its radio jet (GRAVITY Collaboration: Sturm et al., 2018). The data are fitted by a conventional broad-line-region model of a thick disk of gravitationally bound material orbiting a black hole of $3 \times 10^8 M_\odot$. The authors concluded that disk radius is around 150 light days and since earlier a radius of 100 – 400 light days was evaluated previously using reverberation mapping, therefore, these estimates obtained with data analysis if new GRAVITY observations are consistent with previous ones.

5. Constraints on alternative theories of gravity with observations of bright stars near the Galactic Center

5.1. Graviton mass constraints

A theory of massive gravity was introduced by Fierz & Pauli (1939). Later, a number of pathologies such as Zakharov – Veltman – Van Dam – Iwasaki discontinuity, presence of ghosts were found etc. However, in last years theorists created theories of massive gravity without such defects, see recent review on the subject by de Rham et al. (2017). Different ways to evaluate graviton mass were discussed by Goldhaber and Nieto (2010). The LIGO collaboration treated a theory with massive graviton as a feasible alternative theory of gravity and in the first paper where they reported about the first detection of gravitational waves from a merger of two black holes (it was detected on September 14, 2015 and it is called GW150914), about the discovery (Abbott et al., 2016) the team constrained the graviton Compton wavelength $\lambda_g > 10^{13}$ km which could be interpreted as a constraint for a graviton mass $m_g < 1.2 \times 10^{-22}$ eV. Later, this constraint was improved by Abbott et al. (2017a) where the authors found $m_g < 7.7 \times 10^{-23}$ eV from analysis of the GW170104 event signal

From observations of GW signal and corresponding electromagnetic counterparts in different spectral bands from binary neutron star merger GW170817 which represent a kilonova explosion discussed earlier in the literature constraints on speed of gravitational waves from binary neutron star merger have been found $-3 \times 10^{-15} < (v_g - c)/c < 7 \times 10^{-16}$ (Abbott et al., 2017b). Since graviton energy is $E = hf$, therefore, assuming a typical LIGO frequency range $f \in (10, 100)$, from the dispersion relation one could obtain a graviton mass estimate $m_g < 3 \times (10^{-21} - 10^{-20})$ eV which slightly a more worse estimate than previous ones obtained from binary black hole signals detected by the LIGO team (Zakharov et al., 2018a). Assuming Yukawa gravitational potential of a form

$$\Phi_Y(r) = -\frac{GM}{(1+\delta)r} \left[1 + \delta e^{-\frac{r}{\lambda}} \right], \quad (7)$$

size. A very challenging task was arisen to propose a theoretical model for a huge energy release from a very small space region.

and $\delta = 1$ for massive graviton case. Therefore, this representation for gravitational potential can be used to find the lower bound for Compton wavelength λ_g for graviton, i.e. the upper bound for its mass

$$m_{g(\text{upper})} = h c / \lambda_g. \quad (8)$$

In paper by Borka et al. (2013) we found constraints on Yukawa gravity parameters (λ, δ) from observational data on S2 star orbit. Later, we obtained constraints on graviton mass $m_g < 2.9 \times 10^{-21}$ eV from available observational data for S2 star trajectory (Zakharov et al., 2016a) (see also papers by Zakharov et al. (2016b, 2017a,b) for more details). In these considerations we used available data for S2 star trajectory constrain graviton mass. Later, Keck group followed our ideas to improve our estimates with new observational data $m_g < 1.6 \times 10^{-21}$ eV Hees et al. (2017). In paper Zakharov et al. (2018b) we evaluated discovery potential to improve a graviton mass estimate with future observational data for S2 and other bright stars observed with VLT and Keck telescopes, in particular, we evaluated orbital precession for Yukawa potential and obtained an upper limit for a graviton mass assuming that GR predictions about orbital precessions for bright stars moving around the Galactic Center will be confirmed with future observations.

As it was shown by Zakharov et al. (2018b) the longest Compton wavelength could be expressed as

$$\Lambda \approx \frac{c}{2} \sqrt{\frac{(a\sqrt{1-e^2})^3}{3GM}} \approx \sqrt{\frac{(a\sqrt{1-e^2})^3}{6R_S}}, \quad (9)$$

or therefore, after observations of bright stars for several decades an upper bound for a graviton mass could reach around 5×10^{-23} eV.

5.2. Tidal charge constraints

The line element of the spherically symmetric Reissner – Nordström – de-Sitter metric is

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2, \quad (10)$$

where function $f(r)$ is defined as

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{1}{3}\Lambda r^2, \quad (11)$$

M is a black hole mass, Q is its charge and Λ is cosmological constant. In the case of a tidal charge as it was considered by Dadhich et al. (2001), Q^2 could be negative and in this case Q^2 term reflects a presence of an extra dimension. In paper by Zakharov (2018b) it was shown that a total relativistic advance for metric (10) in the first post-Newtonian approximation is

$$\Delta\theta(\text{total}) := \frac{6\pi M}{L} - \frac{\pi Q^2}{ML} + \frac{\pi\Lambda a^3\sqrt{1-e^2}}{M}. \quad (12)$$

and apocenter shift dependences on eccentricity and semi-major axis are the same for Schwarzschild and Reissner – Nordström cases while corresponding factors ($6\pi M$ and $-\frac{\pi Q^2}{M}$) are different, therefore, it is very hard to separate an impact of a tidal charge and black hole mass evaluation uncertainties. For $Q^2 > 0$, there is an apocenter shift due to a presence of the corresponding term in the opposite direction in respect to GR advance. In paper by Zakharov (2018b) bounds in Q^2 and Λ are presented for current and future accuracies for Keck and Thirty Meter telescopes which were discussed by Hees et al. (2017). Similarly to Zakharov (2018b,c) if we adopt uncertainty $\sigma_{\text{GRAVITY}} = 0.030$ mas for the GRAVITY facilities as it was used by GRAVITY Collaboration: Abuter et al. (2018a) ($\delta_{\text{GRAVITY}} = 2\sigma_{\text{GRAVITY}}$) or in this case $\Delta\theta(GR)_{S2} = 13.84\delta_{\text{GRAVITY}}$ for S2 star and assuming again that GR predictions about orbital precession of S2 star will be confirmed with δ_{GRAVITY} accuracy (or $\left|\frac{\pi Q^2}{ML}\right| \lesssim \delta_{\text{GRAVITY}}$), one could conclude that

$$|Q^2| \lesssim 0.432M^2, \quad (13)$$

or based on results of future observations one could expect to reduce essentially a possible range of Q^2 parameter in comparison with a possible hypothetical range of Q^2 parameter in comparison with current and future Keck data.

6. Conclusions

The Particle Data Group (PDG) which is an international consortium of scientists and it collects and reanalyzes results related to the properties of particles, fundamental interaction, astrophysics and cosmology. The PDG issues the Reviews which summarize properties of elementary particles and describe the current status of elementary particle physics, general relativity and cosmology. Our estimate of graviton mass was included in PDG Review update 2019 together with a few other papers where graviton mass constraints are given⁵.

As it was shown that precise observations of bright stars is very efficient tool to check alternative theories of gravity and to investigate a presence of an extended mass distribution near the Galactic Center as it was investigated earlier by Zakharov et al. (2007). We obtained the graviton mass constraints from an analysis of S2 star trajectory and the bounds are consistent and comparable with the constraints presented recently by the LIGO-Virgo collaboration. In our current studies we discuss an opportunities to evaluate parameters of supermassive black hole, stellar cluster and dark matter cloud near the Galactic Center or evaluate parameters of alternative gravity model analyzing apocenter (pericenter) advance after at least one star revolution. However, in the future when astrometric accuracy will be significantly improved one will have a possibility to evaluate a static gravitational potential at the Galactic Center analyzing

⁵<http://pdg.lbl.gov/2019/listings/rpp2019-list-graviton.pdf>.

only small part of stellar orbit similarly to consideration by Kosmo O'Neil et al. (2019), where it was shown that around 40% (or even smaller) of stellar phase coverage is enough for an orbit reconstruction. However, if a contribution of time-dependent component of gravitational potential caused by stellar encounters is significant an orbit reconstruction problem may be more complicated.

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References

- Abbott, B. P., Abbott, R., Abbott, T. D. et al.: 2016, *Phys. Rev. Lett.* **116**, 061102
- Abbott, B. P., Abbott, R., Abbott, T. D. et al.: 2017a *Phys. Rev. Lett.* **118**, 221101
- Abbott, B. P., Abbott, R., Abbott, T. D. et al.: 2017b *Astrophys. J. Lett.* **848**, L13
- Alexander, T.: 2005, *Phys. Rep.* **419**, 65
- Arnold, V. I.: 2000, *Huygens and Barrow, Newton and Hooke. Pioneers in Mathematical Analysis and Catastrophe Theory from Evolvents to Quasicrystals*, (Birkhäuser Verlag, Basel - Boston - Berlin).
- Bardeen, J. M.: 1973, in *Black holes (Les Astres Occlus)*, ed. by C. de Witt and B.S. de Witt (Gordon Breach, New-York – London – Paris, 1973), p. 215.
- Bisnovatyi-Kogan, G. S. & Tsupko, O. Yu.: 2017, *Universe* **3**, 57
- Borka, D., Jovanović, P., Borka Jovanović, V. & Zakharov, A. F.: 2013, *J. Cosm. Astropart. Phys. (JCAP)* **11**, 050
- Broderick A. E. & Loeb, A.: 2006, *Astrophys. J.* **636**, L109
- Broderick, A. E., Fish, V. L., Johnson, M. D. et al.: 2016, *Astrophys. J.* **820**, 137
- Chandrasekhar, S.: 1983, *The Mathematical Theory of Black Holes*, (Clarendon Press, Oxford).
- Cherepashchuk, A. M.: 2016, *Physics-Uspokhi* **59**, 70
- Cherepashchuk, A.M.: 2017, *Astron. Rep.* **61**, 265
- Chu, D. S., Do, T., Hees, A. et al.: 2018, *Astrophys. J.* **854**:12 (10pp)
- Cunha, P. V. P., Herdeiro, C. A. R., Radu, E. & Runarsson, H. F.: 2015, *Phys. Rev. Lett.* **115**, 211102
- Cunha, P. V. P., Grover, J., Herdeiro, C. A. R. et al. : 2016, *Phys. Rev. D* **94**, 104023
- Cunha, P. V. P. & Herdeiro, C. A. R.: 2018, *Gen. Relativ. Grav. D* **50**, 42
- Cunningham, C. T. & Bardeen, J. M.: 1973, *Astrophys. J* **183**, 237
- Dadhich, D., Maartens, R., Papadopoulos, Ph. & Rezania, V.: 2001, *Phys. Lett. B* **487**, 1

- Delva, P., Puchades, N. Schönemann, E. et al.: 2018, *Phys. Rev. Lett.* **121**, 231101
- De Paolis, F., Ingrosso, G., Nucita, A.A. et al. : 2001, *Astron. Astrophys.* **376**, 853
- Do, T., Hees, A., Ghez, A. et al.: 2019, *Science* **365**, 664; arXiv:1907.10731v1 [astro-ph.GA]
- Doeleman, S. S.: 2008, *J. Phys.: Conf. Ser.* **131**, 012055
- Doeleman, S.: 2017, *Nat. Astron.* **1**, 646
- Doeleman, S. S. et al.: 2008, *Nature* **455**, 78
- Doeleman, S. S., Agol, E., Backer, D. et al.: 2009, arXiv:0906.3899 [astro-ph.CO]
- Dokuchaev, V. I. & Nazarova, N. O.: 2017, *JETP Lett.* **106**, 637
- Dokuchaev, V. I. & Nazarova, N. O.: 2019, *Universe* **5**, 183
- Dyson, F. W., Eddington A. S. & Davidson, C.: 1920, *Phil. Trans. R. Soc. London. Series A*, **220**, 291
- EHT Collaboration: Akiyama, K., Alberdi, A., Alef, W. et al.: 2019a, *Astrophys. J. Lett.* **875**, L1 (Paper I)
- EHT Collaboration: Akiyama, K., Alberdi, A., Alef, W. et al.: 2019b, *Astrophys. J. Lett.* **875**, L2 (Paper II)
- EHT Collaboration: Akiyama, K., Alberdi, A., Alef, W. et al.: 2019c, *Astrophys. J. Lett.* **875**, L3 (Paper III)
- EHT Collaboration: Akiyama, K., Alberdi, A., Alef, W. Alberdi, A., Alef, W. et al.: 2019d, *Astrophys. J. Lett.* **875**, L4 (Paper IV)
- EHT Collaboration: Akiyama, K., Alberdi, A., Alef, W. et al.: 2019e, *Astrophys. J. Lett.* **875**, L5 (Paper V)
- EHT Collaboration: Akiyama, K., Alberdi, A., Alef, W. et al.: 2019f, *Astrophys. J. Lett.* **875**, L6 (Paper VI)
- Falcke, H., Melia, F. & Agol, E., 2000, *Astrophys. J.* **528**, L13
- Falcke, H. & Markoff, S. B.: 2013, *Class. Quan. Grav.* **30**, 244003
- Fierz, M. & Pauli, W.: 1939, *Proc. Roy. Soc. Lond. A* **173**, 211
- Gillessen, S, Plewa, P. M., Eisenhauer, F. et al.: 2017, *Astrophys. J.* **837**:30 (19pp)
- Goldhaber, A. S. & Nieto, M. M.: 2010, *Rev. Mod. Phys.* **82**, 939
- de Rham, C., Deskins, J. T. & Tolley, A. J. et al.: 2017, *Rev. Mod. Phys.* **89**, 025004
- GRAVITY Collaboration: Abuter, R., Amorim, A., Anugu, N. et al.: 2018a, *Astron. & Astrophys. Lett.* **615**, L15
- GRAVITY Collaboration: Abuter, R., Amorim, A., Bauböck, M. et al.: 2018b, *Astron. & Astrophys. Lett.* **618**, L10
- GRAVITY Collaboration: Sturm, E., Dexter, J., Pfuhl, O. et al.: 2018, *Nature* **563**, 657
- GRAVITY Collaboration: Abuter, R., Amorim, A., Bauböck, M. et al.: 2019, *Astron. & Astrophys. Lett.* **625**, L10

- GRAVITY Collaboration: Amorim, A., Bauböck, M., Benisty, M., et al.: 2019, *Month. R. Astron. Soc.* **489**, 4606; arXiv:1908.06681v1[astro-ph.GA]
- Hees, A., Do, T., Ghez, A. M. et al.: 2017, *Phys. Rev. Lett.* **118**, 211101
- Herrmann, S., Finke, F., Lülfi M. et al.: 2018, *Phys. Rev. Lett.* **121**, 231102
- Hioki, K. & Maeda, K.: 2009, *Phys. Rev. D* **80**, 024042.
- Holz, D. E. & Wheeler, J. A.: 2002, *Astrophys. J.* **578**, 330
- James, O., von Tunzelmann, E., Franklin P. & Thorne, K. S.: 2015, *Class. Quant. Grav.* **32**, 065001
- Jetzer, P.: 1992, *Phys. Rep.* **220**, 163
- Johannsen, T.: 2016, *Class. Quant. Grav.* **33**, 113001
- Johannsen, T., Broderick, A. E., Plewa, P. M. et al.: 2016, *Phys. Rev. Lett.* **116**, 031101
- Kardashev, N.S.: 1997, *Exper. Astron.* **7**, 329
- Kardashev, N.S.: 2009, *Physics – Uspekhi* **52**, 1127
- Kardashev, N. S., Khartov, V. V., Abramov, V. V. et al.: 2013, *Astron. Rep.* **57**, 153
- Kopeikin, S. M. & Ozernoy, L. M.: 1999, *Astrophys. J.* **523**, 771
- Kosmo O’Neil, K., Martinez, G. D. & Hees, A. et al.: 2019, *Astron. J.* **158**:4 (21pp)
- Luminet, P.: 1979, *Astron & Astrophys.* **75**, 228
- Melia, F. & Falcke, H.: 2001, *Ann. Rev. Astron. Astrophys.* **39**, 309
- Mizuno, Y., Younsi, Z., Fromm, C. M., et al.: 2018, *Nature Astron.* **2**, 585
- Munyaneza, F. & Viollier, R. D.: 2002, *Astrophys. J.* **564**, 274
- Reid, M.: 2009, *Intern. J. Mod. Phys. D* **18**, 889
- Schmidt, M.: 1963, *Nature* **197**, 1040
- Shaikh, R.: 2018, *Phys. Rev. D* **98**, 024044
- Shaikh, R.: 2019, *Phys. Rev. D* **100**, 024028
- Sharov, A. S. & Efremov, Yu. N.: 1963, *Commission 27 of the I. A. U. Information Bulletin on Variable Stars*, Number 23, Konkoly Observatory, Budapest, 18 April 1963.
- Shen, Z. Q., Lo, K. Y., Liang, M.-C. et al. : 2005, *Nature* **438**, 62
- Taton, R. & Wilson, C.: 2003, *The Planetary Astronomy from the Renaissance to the Rise of Astrophysics*, (Cambridge University Press, Cambridge).
- Torres, D. F., Capozziello, S. & Lambiase, G.: 2000, *Phys. Rev. D* **62**, 104012
- Vincent, F. H., Meliani, Z., Grandclément, P. et al. : 2016, *Class. Quant. Grav.* **33**, 105015
- Yakovlev, D. G.: 1975, *Sov. Phys. JETP* **41**, 179
- Young, P.: 1976, *Phys. Rev. D* **14**, 3281
- Zakharov, A. F.: 1986, *Sov. Phys. JETP* **64**, 1
- Zakharov, A. F.: 2014, *Phys. Rev. D* **90**, 062007

- Zakharov, A. F.: 2015, *J. Astrophys. Astron.* **36**, 539
- Zakharov, A. F.: 2017, *J. Phys.: Conf. Ser.* **934**, 012037
- Zakharov, A. F.: 2018a, *Intern. J. Mod. Phys. D* **27**, 1841009
- Zakharov, A. F.: 2018b, *Eur. Phys. J. C* **78**, 689
- Zakharov, A.: 2018c, *Eur. Phys. J. Web Conf.*, **125**, 01010
- Zakharov, A. F.: 2019, *Intern. J. Mod. Phys. D* **28**, 1941003
- Zakharov, A. F., Jovanović, P., Borka, D. & Borka Jovanović, V.: 2016a, *J. Cosm. Astropart. Phys. (JCAP)* **05**, 045
- Zakharov, A. F., Jovanović, P., Borka, D. & Borka Jovanović, V.: 2016b, *Eur. Phys. J. Web Conf.* **125**, 01011
- Zakharov, A. F., Jovanović, P., Borka, D. & Borka Jovanović, V.: 2017a, *J. Phys.: Conf. Series* **798** 012081
- Zakharov, A. F., Jovanović, P., Borka, D. & Borka Jovanović, V.: 2017b, *Eur. Phys. J. Web of Conf.* **138**, 010010
- Zakharov, A. F., Jovanović, P., Borka, D. & Borka Jovanović, V.: 2018a, *Intern. J. Mod. Phys.: Conf. Ser.* **47**, 1860096
- Zakharov, A. F., Jovanović, P., Borka, D. & Borka Jovanović, V.: 2018b, *J. Cosm. Astropart. Phys. (JCAP)* **04**, 050
- Zakharov, A.F., Nucita, A.A., De Paolis F., & Ingrosso, G.: 2005a, *New Astron.* **10**, 479
- Zakharov, A.F., Nucita, A.A., De Paolis F., & Ingrosso, G.: 2005b, *Astron. & Astrophys.* **442**, 795
- Zakharov, A.F., Nucita, A.A., De Paolis, F. & Ingrosso, G.: 2007, *Phys. Rev. D* **76**, 062001
- Zakharov, A.F., Nucita, A.A., De Paolis F. & Ingrosso, G.: 2012, *New Astron. Rev.* **56**, 64
- Zucker, S., Alexander, T., Gillessen, S. et al.: 2006, *Astrophys. J.* **639**, L21