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The uniqueness of determination of the photometric mass ratio of contact binary stars with applications to selected binaries

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Abstract. We have used a Roche-geometry code to pre-generate light curves of contact binary stars using only mass ratio q, fill out f of the Roche lobes, and the inclination i of the orbital plane. We have parametrized *TESS* light curves of several binaries with a trigonometric polynomial. We used our code `UNiQUE` to find the best fit of the overall shape of the observed light curve and its associated parameters (q, f, i). We compared these parameters to those from a rigorous modelling of our selected objects with known mass ratio $q_{\rm sp}$ inferred from spectra. The method is useful to find a close solution before the full modelling. We also got an indication of possible third light present in a system.

Key words: Stars: binaries: eclipsing – Techniques: photometric

1. Introduction

The shape of components of a binary star is dictated by the surface equipotential Ω and the mass ratio $q = M_2/M_1$. A direct way to find q is by measuring the radial velocities of both components and finding the ratio of their semi-amplitudes $q_{\rm sp} = K_1/K_2$. Contact binaries are a special case, where the surface potential $\Omega(q)$ is a function of the mass ratio and is the same for both components. This allows us to connect it to the stellar radii. The system of parameters is more constrained, since the separation of both stars is also connected to their radii. We may define the so-called fill-out factor $f \in \langle 0.0, 1.0 \rangle$ as:

$$f = \frac{\Omega_{inn} - \Omega}{\Omega_{inn} - \Omega_{out}} \tag{1}$$

where $\Omega_{inn}(q)$ and $\Omega_{out}(q)$ pass through the Lagrange points corresponding to the inner (L_1) and outer (L_2) critical surfaces, respectively. Since for contact binaries, we have a monotonous function $q = M_2/M_1 \propto (r_2/r_1)^2$ (see Fig.1 in Hambálek & Pribulla, 2013), for any given f we can estimate the mass ratio from the shape of the light curve (LC).

Another important parameter is the inclination i of the orbital plane of companions from the line of sight of the observer. For contact binary stars, the separation of components is bound to their size, i.e. to their surface potential Ω. Because of this, the minimal inclination angle i_{tot} when a total eclipse is observed is a function of only the mass ratio q.

The amplitude of a light curve, or the difference in depths of primary and secondary minima, is larger for higher inclinations i (and saturated for total eclipses). However, the presence of another body in the system (that may or may not be visible in radial velocities) affects the LC with an added constant flux that results in shallower depths of minima. This leads to smaller apparent inclination values i and further underestimating the photometric mass ratio $q_{\rm ph}$. We can add a new dimensionless parameter - the third light l_3 that can be defined as a ratio of the parasitic flux (F_3) to the overall flux of the contact binary: $l_3 = F_3/(F_1 + F_2)$.

We can describe the overall shape of LC of contact binaries by an orthogonal trigonometric polynomial:

$$I(\varphi) = a_0 + \sum_{k=1}^{n} a_k \cos(2\pi k\varphi) + \sum_{k=1}^{n} b_k \sin(2\pi k\varphi) \tag{2}$$

In this study, we assume only LCs symmetrical around the orbital phase $\varphi = 0.5$ (secondary minimum), i.e. circular orbits and no O'Connell effect of starspots. This allows us to put $b_k = 0$. We have found that for our purpose a polynomial order $n = 10$ is sufficient to represent the most LCs. However, a totally eclipsing system might require the order $n = 20$ (see Rucinski, 1993). If we instead use only polynomial of the order $n = 10$, the residuals in the LC minima of a totally eclipsing system are $\sim$ 5-times larger than those of a partially eclipsing system and are still sufficient for the data noise present in the LC (Hambálek & Pribulla, 2013). We found by comparison that the mass ratio q of such a curve is not sensitive to the higher polynomial solution and opted for the minimum case of $n = 10$.

2. Finding close solutions from *TESS* light curves

We have used the code ROCHE (Pribulla, 2012) to generate LCs of contact binaries with mass ratios $q \in \langle 0.05, 1.00\rangle$, step $\Delta q = 0.025$, fill-out factors $f \in \langle 0.0, 1.0\rangle$, step $\Delta f = 0.25$, and inclinations $i \in \langle 30, 90\rangle$ deg, step $\Delta i = 1$ deg. The Roche potential was used for the shape of the stellar surface. Fluxes were integrated over visible surface elements. This synthetic LC was parametrized by a_k coefficients of eq. 2 ($k = 0..10$). We have generated a library of total 11 895 LCs (see Hambálek & Pribulla, 2013).

For our purpose, the most notable coefficients are: a_1 which affects the difference of depths of primary and secondary minimum, a_2 which (for contact systems) is almost equal to the amplitude of LC, and a_4 which is tied to the

Table 1. Comparison of parameters q, f, i, and l_3 determined from modelling with available spectroscopy and model in literature (subscript L) to those found by `UNiQUE` as the best approximation of the *TESS* light-curve (subscript B). Note: the fill-out factor for EB-type binaries is not applicable.

star	q_L	f_L	i_L [deg]	$l_{3,L}$	q_B	f_B	i_B [deg]	$l_{3,B}$	type
AG Vir	0.341 [a]	0.17 [b]	84 [b]	0.05 [a]	0.325	0.00	78	<0.20	EW A
AW UMa	0.108 [c]	0.30 [c]	78 [d]	0.00 [c]	0.075	0.25	84	0.00	EW
DU Boo	0.206 [b]	0.56 [b]	81 [b]	0.00 [b]	>0.125	0.50	81	0.00	EW A
EL Boo	0.248 [d]	0.00 [e]	74 [e]	1.00 [f]	>0.100	0.00	64	1.00	EW
EQ Tau	0.442 [g]	0.09 [e]	82 [e]	0.00 [g]	0.475	0.00	79	0.00	EW A
FI Boo	0.372 [h]	0.50 [i]	38 [i]	0.30 [h]	0.850	0.75(25)	31	0.30(10)	EW W
FT UMa	0.984 [f]	N/A	60(3) [j]	1.01 [f]	1.000	0.00	61	0.80	EB
SW Lac	0.776 [k]	?	?	<0.05 [k]	0.600	0.25	81	<0.20	EW W
SX Crv	0.066 [g]	?	65(5) [g]	0.00 [g]	0.100	0.25	60	0.00	EW A
V1191 Cyg	0.107 [l]	0.30 [m]	83(2) [m]	0.00 [l]	0.075	0.25	74	0.00	EW W
V523 Cas	0.516 [n]	0.00 [o]	84(1) [o]	0.00 [n]	0.500	0.00	85(3)	0.00	EW W
V753 Mon	0.970 [p]	N/A	75 [q]	0.00 [p]	1.000	0.00	75	0.00	EB
VW LMi	0.423 [a]	0.47 [r]	79 [s]	0.42 [a]	0.325	0.25	71	0.20	EW W
W UMa	0.484 [t]	0.10 [u]	86 [u]	0.00 [t]	0.450	0.00	86(2)	0.00	EW

Source: [a]Pribulla et al. (2006), [b]Pribulla et al. (2011), [c]Pribulla & Rucinski (2008), [d]Pribulla & Rucinski (2006), [e]Deb & Singh (2011), [f]Pribulla et al. (2009), [g]Rucinski et al. (2001), [h]Lu et al. (2001), [i]Christopoulou & Papageorgiou (2013), [j]Yuan (2011), [k]Rucinski et al. (2005), [l]Rucinski et al. (2008), [m]Ekmekçi et al. (2012), [n]Rucinski et al. (2003), [o]Mohammadi et al. (2016), [p]Rucinski et al. (2000), [q]Qian et al. (2013), [r]Sánchez-Bajo et al. (2007), [s]Pribulla et al. (2008), [t]Pribulla et al. (2007), [u]Linnell (1991).

fill-out factor (if component stars are not in contact) and affects the width of minima.

We have written a simple Python code `UNiQUE` to get an arbitrary LC and compare it with those in the pre-generated library. The input LC can be expressed in magnitudes or fluxes. First, it is reduced to a normalized flux (using zero magnitudes of common filters). A Python package `lightkurve` is used to re-bin and phase the LC by the orbital period. We apply a sigma clipping to remove any outliers. If the LC is too sparse, we can run a Savitzky-Golay filter (from `scipy.signal`) on the data to compute a smoothed average curve.

Then the code runs a least-square fitting model (from `lmfit.Model`) to determine the coefficients a_k. Finally, we can compare the set of coefficients to those (a'_k) from pre-computed LCs by computing a difference $D = \sqrt{\sum_k^{10}(a_k - a'_k)^2}$. The code produces a list of similar solutions (i.e. $D <$ an arbitrary value) with parameters q, f, i, and l_3 which were used in their generation. The limiting

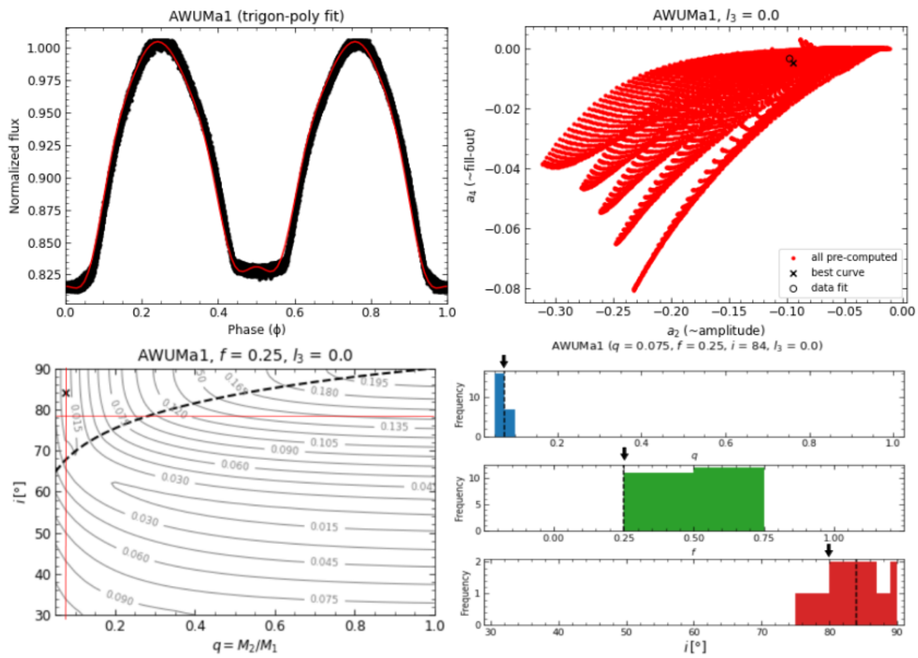

Figure 1. The result for star AW UMa (*TESS* cadence 120 s). Top left: trigonometric fit (red line) of observed LC (black). Top right: the best solution ($\times$) with $l_{3,\mathrm{B}} = 0.0$ (even if unknown was assumed) is close to a pre-computed LC ($\circ$). Bottom left: Best solution ($\times$) in difference plot q vs. i corresponds to total eclipses (above the dashed line). Bottom right: Colour histograms (q - blue, f - green, i - red) of similar values with the smallest differences. The solution from rigorous modelling is marked with arrows, while the best result of `UNiQUE` is represented by a vertical dashed line.

value of D is selected based on the minimum value of D_{min} and a fraction of the standard deviation. The number of contours (Fig. 1 bottom left) depends on the total span of all D values. A statistical distribution is also generated to see the most probable result (Fig. 1 bottom right).

If the third light l_3 is known, the user can compute the LC coefficients for this specific value only. Otherwise a default set of typical values of $l_3 \in \{0.0, 0.2, 0.4, 0.6, 0.8, 1.0\}$ is used as a proxy for an unknown amount of the third light. In general, this increases the uncertainty of mass ratio q, which can be then treated only as a lower limit.

We have selected 14 eclipsing binaries with known q_{sp} and found photometric Roche-model parameters f, i, and possible l_3 in literature. In Table 1, we compare them to the best results from the `UNiQUE` library of LCs. All *TESS* LCs were analyzed by individual sectors. Sometimes, the results found a range of "best" parameters caused mainly by uneven maxima brightness - the so-called O'Connell effect (see e.g. Wilsey & Beaky, 2009). This leads to higher values of i and lower values of q. Totally eclipsing systems (e.g. V523 Cas, AW UMa, etc.) have a small uncertainty of q, but virtually all values of i above the dashed line like in Figure 1 bottom left are indistinguishable. We note that the close binary AW UMa is accompanied by a third component on a $\sim$ 17-year orbit (Pribulla & Rucinski, 2006) that is not affecting the LC and thus the estimation of close binary parameters q, i, f. Also, systems FI Boo, SW LAc, V753 Mon, and W UMa have wider third components (Pribulla & Rucinski, 2006) on longer orbits than $\sim$ 20 years. Furthermore, VW LMi is a quadruple system (Pribulla et al., 2006) with the second close binary being non-eclipsing, thus affecting only l_3. EB-type systems (FT UMa and V753 Mon) are not in contact and our `UNiQUE` solutions are strictly favouring only LCs with $f = 0$. In general, l_3 was kept a free parameter and in our test cases, it still was sensitive to values as small as $l_3 \sim 0.05$ (e.g. SW Lac) when it selected an LC with the nearest non-zero l_3.

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