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Binary Cepheids: Insights from a simulation-based approach

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Abstract. The growing evidence that the majority of stars are born in binary or multiple stars is causing a paradigm shift in our thinking about Cepheids from single to binary and possible multiple systems, and calls for action to include the presence and influence of companion(s) in the theories of stellar evolution and pulsations. Companions might alter every parameter of pulsating stars, from their apparent magnitude, to their pulsation cycles and internal structure, including mass gain/loss. These changes – to some degree – should affect every Cepheid, and anticipating them through simulations, as well as discovering them though observations, is paramount for refining models of stellar pulsations and gaining detailed knowledge about pulsation mechanisms. I present the current state of knowledge regarding binary Cepheids, discuss the strengths and limitations of various detection techniques, and explore how simulations can help observations by providing insights that are currently unattainable through observational means alone.

Key words: astronomical simulations – Delta Cepheid variable stars – binary stars – Cepheid distance

1. Introduction

Classical Cepheids (hereinafter Cepheids) are radially pulsating stars, widely used in astrophysics to determine distances and constrain theoretical models of stellar evolution and pulsations. They are such potent sources of information because of their special location on the Hertzsprung-Russell diagram inside the instability strip confined by the luminosity ($2 - 5\,L_\odot$) and effective temperature ($4000 - 7000$ K), which drive their pulsations. As Cepheids' outer layers expand and contract, causing periodic changes in the physical parameters (radius, effective temperature, surface gravity, and luminosity), Cepheids' brightness also changes periodically, with periods ranging between about 2 and 100 days, correlating with their mean luminosity. This period-luminosity (P-L) relation (Leavitt Law, Leavitt & Pickering, 1912) renders Cepheids essential for accurate determinations of extragalactic distances and the Hubble constant (Riess et al., 2022).

Metallicity is another important factor that affects the Cepheid mean luminosity in the sense that metal-rich Cepheids are systematically brighter than their metal-poor counterparts (Breuval et al., 2022). The effect of metallicity is preserved in the shape of Cepheid light curves and because of that the information about the metalllicity can be retrieved from the photometric observations alone (Hocdé et al., 2023) when spectroscopic observations are inaccessible. In fact, metallicity, together with luminosity, effective temperature, and other parameters that describe the internal structure of Cepheids and govern their evolution (e.g. overshooting, rotation), are implemented in the newest generations of codes for the light curve modeling (e.g. De Somma et al., 2022). Such modeled light curves – when compared to the observed ones – give insight into physical features of Cepheids that could not be examined otherwise, and advance our knowledge about these pulsators, which turns them into even more precise tools for distance determination.

This advancement has been built upon the assumption that Cepheids are single stars. However, recent theoretical and empirical studies concur that at least 60 – 80% of Galactic Cepheids are in fact binary stars (Szabados, 2003; Mor et al., 2017; Kervella et al., 2019a), and a significant percentage of them (30 – 40%) are in multiple systems (Evans et al., 2005; Dinnbier et al., 2024).

Binary Cepheids, especially in eclipsing systems, are highly desired configurations, because they make unique stellar laboratories where stellar parameters (particularly masses and radii) of each companion can be determined independently and very accurately, and next used to further constrain stellar and pulsation models. An example of the remarkable utility of Cepheids in binary eclipsing systems is the determination of the dynamical mass of the Cepheid OGLE-LMC-CEP0227 with an unprecedented accuracy of 1% (Pietrzyński et al., 2010), which resolved the long-standing mass discrepancy problem (Keller, 2008), i.e. the discrepancy between Cepheid masses derived from stellar and pulsational models.[1] Pietrzyński et al. (2010) found that the dynamical mass of OGLE-LMC-CEP0227 agrees with the pulsational mass, thus challenging evolutionary models to identify the missing processes responsible for producing Cepheids that would reconcile this mass discrepancy.

Unfortunately, the overwhelming majority of known Cepheids remain categorized as single stars. In the Milky Way (MW) out of more than 3000 classical Cepheids (Pietrukowicz et al., 2021) about 200 have confirmed companions (Szabados, 2003; Kervella et al., 2019a; Kervella et al., 2019b; Shetye et al., 2024), which constitutes only about 6% of all known MW Cepheids. In the Small and Large Magellanic Clouds (SMC, LMC) out of about 10,000 Cepheids Soszyński et al. (2015) known are only about 70 Cepheids in binary systems (Szabados & Nehéz, 2012; Pilecki et al., 2021, 2024), which accounts for less than 1%. Such a gap between the percentage of observed binary Cepheids and the expected ones

[1] This discrepancy is about 20 – 30% in the sense that the pulsational masses are systematically smaller than the evolutionary ones.

(60 − 80%) indicates a strong selection bias due to the limitations of various detection techniques.

For example, binary Cepheids reported by Pilecki et al. (2021) all have companions that are giant stars. In fact, the large brightness of their companions that makes these Cepheids appear brighter than expected and thus above the P-L relation was the reason these stars were chosen for the spectroscopic analysis in the first place (see Fig. 1). A different criterion allowed Evans (1992) to recognize binary Cepheids with blue and hot main sequence companions due to larger than expected brightness in the ultraviolet domain; however, such observations could only be made from space and for the brightest of Cepheids ($V < 8$ mag). Binary Cepheid candidates can also be selected through the analysis of their photometric amplitudes in different passbands (Klagyivik & Szabados, 2009); especially a high A_V/A_B ratio can indicate a blue companion (either physical or on the line of sight) because the light from the blue companion reduces A_B more severely than A_V. Additionally, radial velocity amplitudes ($A_{\rm RV}$) of Cepheid + companion pairs are larger than single Cepheids due to the $A_{\rm RV}$ of orbital motion superimposed on the $A_{\rm RV}$ of pulsation, and can be another indicator of binarity through the parameter $A_{\rm RV}/A_B$, which should be larger for binaries than for single Cepheids (Klagyivik & Szabados, 2009). Another two methods allowed for detection of angularly resolved binary Cepheids due to the common proper motion of Cepheid + companion pairs (Kervella et al., 2019a) and due to the anomaly in Cepheids' proper motion (Kervella et al., 2019b).

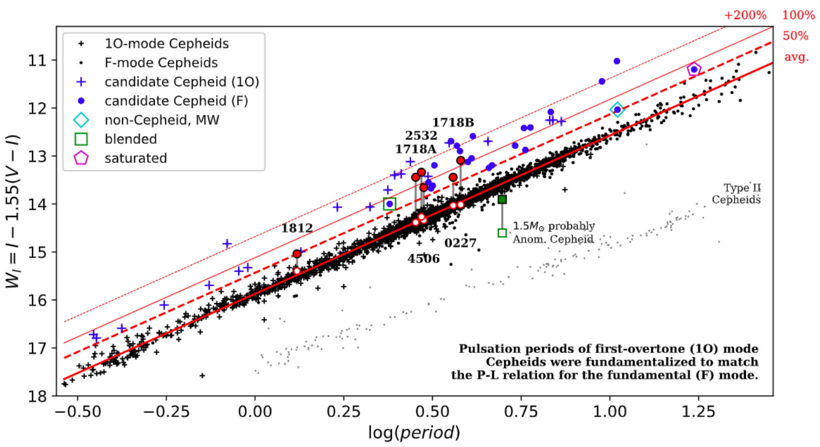

Figure 1. The P-L relation for the reddening-free Wesenheit index. Confirmed binary Cepheids are marked as red circles, while the candidates for binary Cepheids – as blue circles and plus signs. Figure excerpted from Pilecki et al. (2021); for a full description the reader is referred to the source publication.

The variety of methods and their applications demonstrate that no single technique can reliably detect binary Cepheids with all types of companions, while the gap between the expected and actual detections of binary Cepheids suggests that all the current methods combined are insufficient for identifying binary Cepheids with all possible companion types. This gap also highlights a 'blind spot' in our understanding of Cepheid evolution – which may predominantly occur in binary or multiple systems – and of Cepheids' characteristics, which might be affected – or even shaped – by interactions with their companion(s). In order to bridge this gap we resort to simulations of binary Cepheids.

2. Insights from binary population synthesis methods

Binary population synthesis (BPS) codes (e.g. Izzard et al., 2006; Belczynski et al., 2008) rely on approximate formulas to evolve stars from the main sequence until the compact object stage[2], enabling the simulation of vast populations (hundreds of thousands) of binary systems over billions of years. While BPS codes are computationally efficient, they simplify evolutionary processes and omit more complex interactions, making them tools suited for deriving statistical properties of large populations of binaries rather than studying individual systems in detail.

Synthetic populations of binary Cepheids are free from the selection bias inherent to limitations of the observing techniques. They allow us to explore the full range of physical and orbital characteristics that Cepheids and their companions can exhibit as they evolve together (orbital periods, eccentricities, mass ratios, companion types) – only a small fraction of which is accessible through observations. Moreover, synthetic populations are unaffected by reddening, a factor that significantly influences distance determinations. Additionally, while reddening changes Cepheids' apparent color to more red, hot main-sequence companions shift it to more blue, resulting in a complex, poorly understood combined effect. The synthetic population enables to separate the two factors by eliminating the reddening effect altogether.

The BPS codes can be executed with different values/distributions of input parameters: metallicity, star formation history, initial mass function, binary fraction, etc., resulting in many variants of synthetic populations. By comparing these variants one can explore the effects that each parameter has on the evolution and orbital/physical characteristics of binary Cepheids. This analysis can be performed by altering one parameter at a time, an approach achievable only through simulations, as the observed properties of binary Cepheids result from the complex interplay of all parameters.

[2]More specifically, the final stage of evolution can be a compact object (a white dwarf, a neutron star, or a black hole), or a supernova remnant, which is not a compact object but is the final stage of star's evolution.

Below I present a subjective compilation of findings regarding classical Cepheids in binary systems that were made possible thanks to the BPS approach, presented in Karczmarek et al. (2022) and Karczmarek et al. (2023). Simulations of binary Cepheids made by other authors (Neilson et al., 2015; Dinnbier et al., 2024) and simulations of other types of classical pulsators in binary systems, such as anomalous Cepheids (Gautschy & Saio, 2017), metal-rich RR Lyrae (Bobrick et al., 2024), and ultra-low-mass pulsators exhibiting RR-Lyrae pulsations (Karczmarek et al., 2017), although beyond the scope of this paper, are mentioned for the sake of completeness.

2.1. Effect on the extragalactic distance scale

Binary Cepheids with giant companions are easily detectable as outliers above the P-L relation (see Fig. 1). For the purpose of distance determinations using the Cepheid P-L relation these outliers are always removed so that their presence does not shift the P-L relation towards brighter magnitudes. However, the remaining binary Cepheids with companions that are not bright enough to be classified as outliers can still contribute to the shift of the P-L relation, just with a lesser effect. Given that 60 − 80% of Cepheids have companions, which add extra light to the Cepheid apparent brightness, the majority of Cepheids would reside slightly above the P-L relation, shifting it towards brighter magnitudes. Their compound contribution can result in a non-negligible shift that cannot be estimated from observations alone because such binary Cepheids cannot be distinguished from the single ones and properly addressed. This effect has long been assumed in the literature as negligible in the context of distance determinations, but a quantification of this phenomenon was done only recently by Karczmarek et al. (2023).

The authors used the BPS code StarTrack (Belczynski et al., 2008) to evolve 200,000 binary systems with masses between 2.5 and 12 $M_\odot$ and metallicities Z = 0.004, 0.008, 0.02, which correspond to the metallicites of Cepheids in the SMC, LMC, and MW, respectively. By varying other BPS parameters they created a total of 48 variants of synthetic populations, which allowed them to assess the most probable configuration of parameters, i.e. the one that reflects best the observed characteristics of Cepheids. The effect of binarity was controlled by the binarity percentage, $f_{\rm bin}$, which was arbitrarily set to one of 5 values: 0% (only single Cepheids), 25%, 50%, 75%, 100% (only binary Cepheids).

The authors calculated the changes in the slope and the zero point of the P-L relations in various passbands (from B to K, including Wesenheit indices) for populations of Cepheids with different binarity fractions, taking as a reference point the population with $f_{\rm bin} = 0\%$. They reported that the shifts in the P-L slopes due to binarity are negligible within errors, but the shifts in the zero points can be as severe as 0.05 mag (the shift in the B-band for $f_{\rm bin} = 100\%$). This result however needs to be put in the context of cosmic distance determinations. The distance derived using the P-L method is in fact a difference between two zero points of the P-L relations, calculated for a reference and target galaxy (i.e. the distance modulus, μ). Therefore the final shift in the zero points needs to take into account the binarity percentage in the reference and target galaxies. The final shift will be the largest if the $f_{\rm bin}$ of the reference galaxy is drastically different from the $f_{\rm bin}$ of the target galaxy, and vice versa. If the $f_{\rm bin}$ of the two galaxies is the same, a very small shift is possible due to the population effects (e.g. different metallicities). The matrices in Fig. 2 show all the possible shifts in distance moduli, $\Delta\mu$, between the target and reference galaxies for the V and K bands. The conclusion of this study is that the effect of binarity depends on the passband of the P-L relation used for the distance determination, and is the smallest when the binarity percentages in the reference and target galaxies are similar.

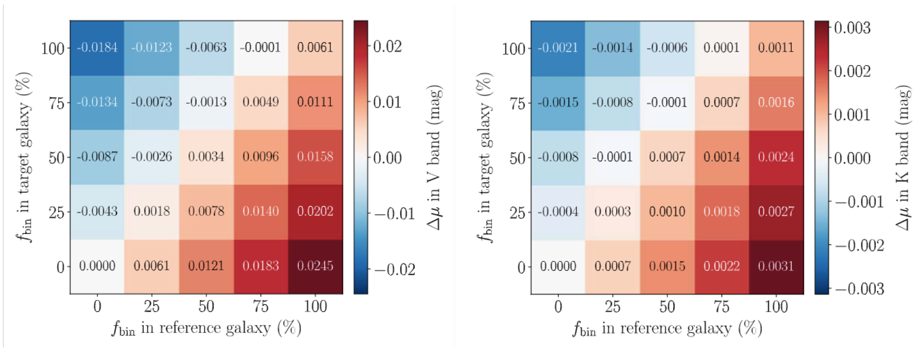

Figure 2. V- and K-band grids of possible values of distance modulus shifts, $\Delta\mu$, as a function of the binarity percentage in the reference (Z = 0.008) and target (Z = 0.02) galaxies. Figure excerpted from Karczmarek et al. (2023); for a full description the reader is referred to the source publication.

In the Wesenheit index W_H, $\Delta\mu$ ranges from -0.004 to 0.004 mag. If we introduce a random shift from this range to the distance moduli of all 37 galaxies hosting supernovae type Ia and Cepheid variables (Riess et al., 2022), we will not shift the value of the Hubble constant, but we will increase its spread by $0.07\,{\rm km\,s^{-1}\,Mpc^{-1}}$ or 0.1%. This result is indeed negligible for the current goal of achieving the 1% precision of the Hubble constant but might become relevant in the future in pursue of even higher precision.

2.2. Relation between the mass ratio and magnitude difference

The magnitude difference, also known as contrast, is the difference in brightness between the Cepheid[3] and its companion. The relation of the magnitude difference to the mass ratio (companion to Cepheid) was first explored by Anderson & Riess (2018), not through the BPS method but assuming a typical Cepheid of a 20-day pulsation period and accompanied by a main-sequence companion. They showed that the contrast decreases as the mass ratio increases and is the smallest for the mass ratio equal to 1. Secondly, the contrast is systematically larger in the H band than in the V band (i.e. Cepheids' companions are fainter in the H band) which is expected since the companions are main-sequence stars and therefore are systematically hotter and bluer than Cepheids. This relation between the mass ratio and contrast was further explored and expounded by Karczmarek et al. (2022) who used the BPS code StarTrack with parameters as described in section 2.1. They found a satisfactory agreement with Anderson & Riess (2018)'s results (see Fig. 3); additionally they presented for the first time the mass ratio-contrast relation for binary Cepheids traversing the instability strip while in the Hertzsprung gap (the co-called first crossing) and for binary Cepheids with giant companions. Comparison with the empirical data hints to the possibility that some of the observed binary Cepheids might be in fact first crossers as they seem to reside in Fig. 3 below the trend created by the 'blue loopers' (i.e. Cepheids on their second and third crossing).

Another interesting application of the mass ratio-contrast relation is that the mass ratio can be estimated from the knowledge of the contrast between the Cepheid and its companion, which in turn can be obtained from high-precision interferometric measurements (e.g. Gallenne et al., 2019).

2.3. Percentage of binary Cepheids in the Large Magellanic Cloud

The binarity percentage among MW Cepheids (60 – 80%) presented in section 1 might not be a universal value. Observational data alone are not sufficient to estimate the percentage of binaries among Cepheids in other galaxies (e.g. the LMC or SMC). However, they can help estimating this percentage in tandem with the BPS method. Karczmarek et al. (2022) put together two pieces of information: the number of Cepheids in eclipsing binaries in the LMC observed during 29 years of the Optical Gravitational Lensing Experiment project (OGLE, Soszyński et al., 2008; Pilecki et al., 2018) and the number of binary Cepheids with giant companions from their synthetic population,[4] which constitute 3 – 5% of the entire synthetic population of binary Cepheids. By comparing

[3]This difference vary as the Cepheid changes its brightness during the pulsation cycle. A simple approximation of the contrast value is made by assuming a fixed brightness of a Cepheid, equal to its mean brightness.

[4]In order to compare both numbers, the number of synthetic binary Cepheids with giant companions had to be trimmed to account for orbital periods below 29 yr and favorable inclination angles $i \geq 83$ deg that allow detection through eclipses.

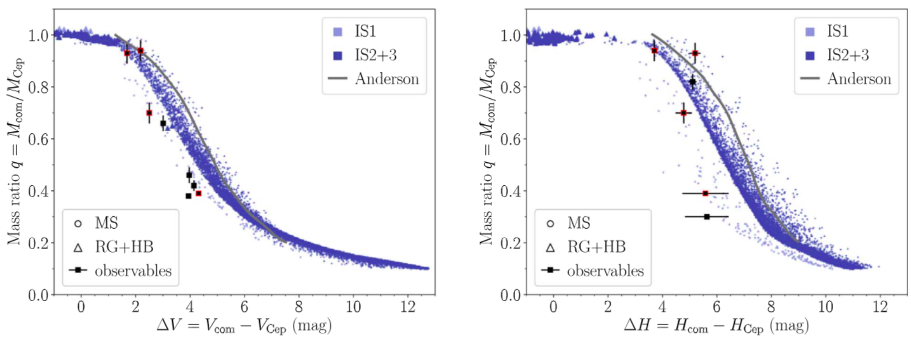

Figure 3. Magnitude difference (contrast) between MW Cepheids and their companions in the V (left) and H (right) bands vs. mass ratio. Empirical data are marked as black squares; systems present on both panels are additionally marked in red. Figure excerpted from Karczmarek et al. (2022); for a full description the reader is referred to the source publication.

the two numbers to each other and to the total number of Cepheids known in the LMC (Soszyński et al., 2015) Karczmarek et al. (2022) estimated the percentage of binary Cepheids in the LMC to be at least 55%, and most likely much higher, i.e. closer to 100%.

3. Beyond binaries – prospects in simulation-based approaches

The extra light contributing to the apparent brightness of Cepheids, discussed extensively in section 2.1, does not have to come from a gravitationally bound companion but from a blend, i.e. a star that coincidentally lies along the Cepheid's line of sight. Because of that all Cepheids with additional light are considered *candidates* for binary stars until their binarity is confirmed through spectroscopy or astrometry. However, these follow-up observations are highly time-consuming and may ultimately reveal that the Cepheid is merely a blend and not an exciting new discovery of a binary system. Therefore, an effective pre-selection of binary candidates is crucial to achieving a higher confirmation rate of binary Cepheids in subsequent follow-up studies.

A promising method to detect additional light in the apparent brightness of Cepheids coming from blue and hot stars was proposed by Madore (1977). As a Cepheid undergoes its pulsation cycle, its effective temperature and luminosity change periodically, leading to corresponding changes in the Cepheid's color. These variations can be represented on the color-color diagram of $U - B$ versus $B - V$, as a loop-shaped pattern that closely resembles a descending straight line. When contaminated with additional light coming from a hot blue star, the loop deforms into more open and horizontal (see Fig. 4). The shape of the deformation holds information about the effective temperature and luminosity of the contaminating star, but is insufficient to determine whether the contaminator is gravitationally bound to a Cepheid. This piece of information can be obtained from a synthetic population of binary Cepheids. If the contaminator lies on the Hertzsprung-Russell diagram in the location frequently occupied by Cepheid companions, it is more probably a true Cepheid companion rather than a blend.

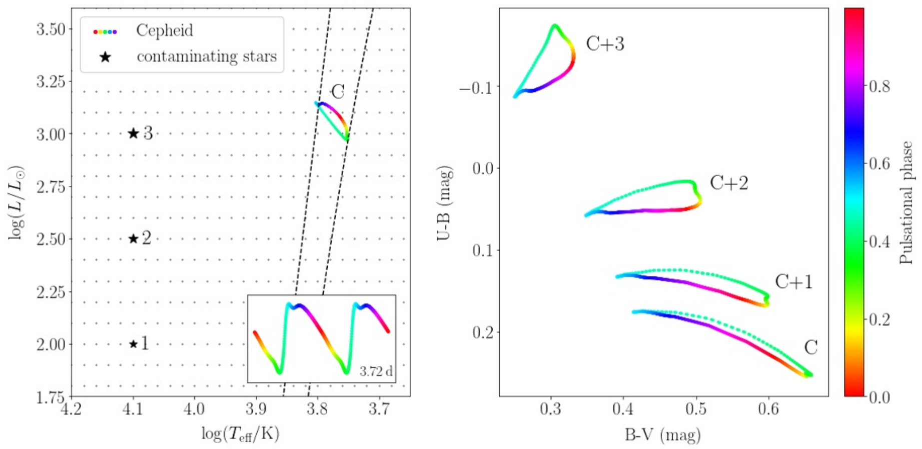

Figure 4. The location of a Cepheid (colorful loop) and three hotter constant stars (black stars) on the Hertzsprung-Russell diagram (left) is transformed on the color–color diagram (right), where the loops labeled as C+1, C+2, C+3 are deformed due to the contamination light coming from the stars 1, 2, 3, respectively.

Last but not least, a growing number of binary Cepheids identified today are often found to be either triple systems (Anderson et al., 2015) or binaries in which the Cepheids themselves resulted from mergers (Neilson et al., 2015; Pilecki et al., 2022) – indicating an origin in triple systems as well. The similar conclusion was reached by Dinnbier et al. (2024) who conducted the state-of-the-art n-body simulations of star clusters of various masses and densities, embedded in a galaxy, and consisting only of binary systems. They showed that mergers are very common among Cepheid progenitors, and prevent about 30% of them from evolving into Cepheids. On the other hand, one in five Cepheids forms as a result of a merger between stars with masses below the minimum threshold required for the Cepheid stage, meaning that had they not merged, they would have never become Cepheids. The binary percentage that Dinnbier et al. (2024)

derived from their n-body simulations is 42%, and underestimates the observed binary Cepheid percentage (60 – 80%) by approximately a factor of 2. This suggests that Cepheid progenitors should rather be formed in triple systems, not binaries.

Observations and simulations of binary Cepheids continue to surprise us with discoveries that call for the paradigm shift in our thinking about Cepheids from single to binary to triple and possibly multiple systems. Some simulations and models can continue with approximating Cepheids as single stars, especially when their objectives require using near-infrared passbands or/and cleaning the sample from outliers. Other ones will have to be adjusted to include the effect of companions in the calculations, as neglecting their presence might lead to inaccurate estimations of Cepheid features. The outcomes of BPS models might to some extent help to determine when single-star approximations remain valid and when multi-star simulations become necessary. The outcomes might also guide future observational campaigns, suggesting optimal observational strategies and facilitating the identification of new binary systems. The results of BPS studies and of other simulation-based approaches help us to broaden our knowledge about the formation and evolution of Cepheids and might prove essential in addressing long-standing Cepheid-related problems, such as the mass discrepancy, age discrepancy, or wind-induced mass loss.

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