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Are binary-star populations regionally different?

– in memory of Sverre Aarseth –

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Abstract. For synthesising star clusters and whole galaxies, stellar populations need to be modelled by a set of four functions that define their initial distribution of stellar masses and of the orbital properties of their binary-star populations. The initial binaries are dynamically processed in different embedded clusters explaining differences in the observed populations. The approach summarised here, for which the Aarseth Nbody codes have been instrumental, allows inference of the initial conditions of the globular cluster ω Cen and the quantification of the stellar merger rate as a function of stellar spectral type, of the role of multiples and mergers for the Cepheid population, and predictions of extragalactic observables. The observability of the four initial distribution functions and their physical and philosophical meaning are also briefly raised. Evidence for the variation of these functions on the physical conditions of star formation and future steps towards extensions to include higher-order multiple systems are touched upon.

Key words: stellar populations – initial binary star population – ω Cen – Cepheids – stellar mergers

1. Introduction

The primary distribution function defining the appearance as well as the dynamical and elemental-abundance evolution of a stellar population is the stellar initial mass function (IMF, $\xi(m)$). But what about specifying how the stars are distributed in binary or higher-order multiple systems? These provide star-relevant internal degrees of freedom that affect the stellar-dynamical and astrophysical evolution of stellar populations.

Concerning the stellar IMF, the number of stars with masses in the interval m to $m + \mathrm{d}m$ born in one embedded cluster is $\mathrm{d}N = \xi(m)\,\mathrm{d}m$. Its dependence on the metal abundance and density of the embedded cluster is reasonably well known (Kroupa *et al.*, 2024). The embedded clusters form in molecular cloud clumps with a star-formation efficiency (SFE) near 30 per cent with half-mass radii $r_{\rm h} < 1\,{\rm pc}$ (Marks & Kroupa 2012; Zhou, Kroupa & Dib, 2024) and masses in stars, $M_{\rm ecl}$, ranging from about a dozen stars to many millions of stars, depending on the properties of the hosting molecular clouds as set by the host-galaxy's state (e.g. in self-regulated equilibrium or interacting). Observations of nearby molecular cloud clumps with ongoing star formation suggest the SFE to lie between a few and 30 per cent (e.g. Lada & Lada, 2003, fig. 25 in Megeath *et al.*, 2016, fig. 8 in Zhou, Dib & Kroupa, 2024) while Nbody calculations of the very well observed $1-2\,$Myr old Orion Nebula Cluster (mass $\approx 10^3\,M_{\odot}$), the NGC3603 very young cluster (mass $\approx 10^4\,M_{\odot}$), and the star-burst cluster R136 (mass $\approx 10^5\,M_{\odot}$) suggest SFE $\approx$ 33 per cent in all cases (Kroupa, Aarseth & Hurley, 2001; Banerjee & Kroupa, 2018). A SFE near 33 per cent may be the characteristic effective SFE involved in an embedded cluster emerging from a molecular cloud clump. The region-wide or galaxy-wide IMF (gwIMF) follows by adding all stellar IMFs of all formed embedded clusters in the region, leading to the gwIMF evolving with the metal abundance and gas density (Jerabkova *et al.*, 2018).

Can a similar description of the binary-star properties of different stellar populations or galaxies be arrived at? A problem facing this goal is that no successful star-formation theory can be resorted to in order to perform this synthesis, since at present the simulations of star formation have neither achieved sufficient realism to match the small embedded clusters in the nearest star-forming cloud in Taurus-Auriga nor Orion Nebula type clusters (for a discussion see Kroupa *et al.* 2024).

2. Are stars born in binary or in higher-order multiple stellar systems?

Owing to the conservation of angular momentum pre-stellar molecular cloud cores within a clump form not one star but multiple stellar systems. These have stellar-dynamical decay time-scales of $10^4 - 10^5\,$yr leaving, within this time-scale, a population consisting of 50 or 33 per cent binaries and 50 or 66 per cent single stars if all cores were to spawn non-hierarchical triple or quadruple systems, respectively. Because $\approx 1\,$Myr old populations in low-density star-forming regions have a binary fraction near 1 this must imply that the vast majority of all stars form as binaries or hierarchical, dynamically stable higher-order systems (Goodwin & Kroupa, 2005). The fraction of higher-order multiple systems is sufficiently small (Goodwin *et al.*, 2007; Duchêne & Kraus, 2013; Offner *et al.*, 2023; Merle, 2024) to be neglected in deriving the relevant distribution functions. The above is true for late-type stars less massive than a few $M_{\odot}$ but more massive stars appear to be born with a triple and quadruple fraction near to unity (Offner *et al.*, 2023). Thus, it may be assumed to sufficient approximation that the total fraction of multiple systems is $f_{\rm mult,birth} = 1$ at birth. Here

$$f_{\rm mult} = N_{\rm mult} / \left(N_{\rm sing} + N_{\rm mult} \right) \;, \tag{1}$$

where $N_{\rm mult}$ is the number of multiple systems and $N_{\rm sing}$ is the number of single stars in a star-count survey. For example, if we survey 100 main-sequence systems on the sky and 50 are binaries, 15 triples and 5 quadruples then $f_{\rm mult,ms} = 0.7$, while the observer misses 95 stars if none of the multiple systems are resolved. Triples and quadruples can thus be counted as binaries, setting $f_{\rm bin} \approx f_{\rm mult} = 0.7$ in this example.

3. The distribution functions

Given that late-type stars (massive stars obey different pairing rules, see e.g. Dinnbier *et al.*, 2024; the birth orbital-parameter distribution function defining massive stars still needing to be formulated) appear to form as binaries, secondarily to the stellar IMF, three additional birth distribution functions are needed to specify the dynamical internal degrees of freedom of a just born stellar population: the period distribution function, $f_{\rm P,birth}(P)$, the mass-ratio distribution function, $f_{\rm q,birth}(q)$ ($q = m_2/m_1 \leq 1$ with m_1, m_2 being the mass of the primary and secondary, respectively), and the eccentricity distribution function, $f_{\rm e,birth}(e)$. $\mathrm{d}N = f_{\rm x,birth}(x)\,\mathrm{d}x$ is the number – or fraction amongst all systems in a survey – of binary systems with orbital element x in the range x to $x + \mathrm{d}x$, with $x = P, q, e$. Note that P is customarily in days. If $a_{\rm AU}$ is the orbital semi-major axis in astronomical units then by Kepler's third law, $m_{\rm sys} = m_1 + m_2 = a^3_{\rm AU}/P^2_{\rm yr}$, where $P_{\rm yr} = 365.25\,P$ is the orbital period in years.

The following questions need to be answered: what are the mathematical forms of $f_{\rm P,birth}, f_{\rm q,birth}, f_{\rm e,birth}$, and if these exist, are these as dependent on the physical conditions of the gas clouds as is $\xi(m)$ (see "The Universality Hypothesis" in Kroupa 2011)? Answering these questions is not trivial because none of the distribution functions are observable: at any time a population of stars is surveyed, it is already at an evolved astrophysical stage (massive stars have evolved) and in an evolved stellar-dynamical state because stars begin to gravitationally interact in the dense embedded clusters as soon as they become ballistic particles. The properties of the binary population thus changes on the crossing time scale of the embedded cluster which is $\lesssim 0.3\,$Myr. The initial distribution functions thus never exist physically, but they are required for mathematically modelling stellar populations. They are "hilfskonstrukts" (Kroupa & Jerabkova, 2018).

For example consider a very low-mass embedded cluster in which a handfull of (say 4) binaries are observed to have formed within a clump with a half-mass radius $r_{\rm h} \approx 0.1\,$pc. With an average stellar mass of $0.3\,M_\odot$ and a star-formation efficiency of 1/3 the clump mass becomes $M_{\rm cl} \approx 8\,M_\odot$ such that it's crossing time is $t_{\rm cross} = 2\, r_{\rm h}/\sigma \approx 0.33\,{\rm Myr}$, with

$$\sigma \approx \left(G\, M_{\rm cl}/r_{\rm h}\right)^{1/2} , \tag{2}$$

being the nominal velocity dispersion and G the gravitational constant. Any wide binary that forms in such a low-density molecular cloud clump, i.e. a very low-mass embedded cluster, will thus have experienced a number of mild encounters with other binaries at an observation time of $0.5 - 1$Myr, with (in this case very mild) stellar-dynamical ejections likely having removed a few stars already by this time (Kroupa & Bouvier, 2003). More massive embedded clusters are more stellar-dynamically active therewith dynamically processing the initial population on a shorter time scale (Kroupa, 1995a).

Important note: In order to not affect the stellar IMF it is of paramount importance, when constructing an initial stellar population, to first create an array of stellar masses from a given $\xi(m)$ and to then pair stars chosen from this array into binaries or higher-order multiples such that some $f_{\rm q}$ is obeyed (Oh & Kroupa, 2016; Dinnbier *et al.* 2024; Dvorakova *et al.*, 2024).

Fig. 1 demonstrates the problem: About 1 Myr old populations in star-forming regions of low density have a significantly higher binary fraction than the main sequence (ms) stars in the Galactic field which have an average age of about 5 Gyr. This difference in the binary fractions, $f_{\rm bin,birth} \approx 1$ vs $f_{\rm bin,ms} \approx 0.5$, is readily explained through the disruption of binaries in the embedded clusters in which they formed through stellar-dynamical encounters.

Before describing the procedure how the birth distribution functions were arrived at a historical note is useful to appreciate the developments: By the late 1980s it was already well known that (i) binary systems form through three-body encounters in star clusters and these slow down the Nbody computations significantly, especially if multiple binary systems interact in the core of the clusters. And (ii) surveys of field stars were showing many of them to be binary systems (e.g. Abt 1983) such that some initial binary population appeared to be necessary when initialising the Nbody models. Therefore Sverre Aarseth spent much effort developing integration, transformation and regularisation methods to allow his Nbody codes to computationally handle such multiple systems while the stars evolve astrophysically. But the link between star clusters being the origin of the field population was not significantly entertained (e.g. Wielen 1971). In late-1992, a very excited Sverre Aarseth approached me at coffee time at the Institute of Astronomy in Cambridge and truly dragged me to his office to explain the newly-found implementation of chain regularisation into the Nbody code (later published in Mikkola & Aarseth, 1993). I was not able to follow the technical details of his explanations (being initially preoccupied with a cookie that had disagreeably melted into my coffee cup and with merely counting stars to obtain $\xi(m)$), but it dawned on me that this discovery improved his Nbody codes dramatically, making the Nbody integration of star clusters with a large initial binary population even more accessible (his codes were the most advanced

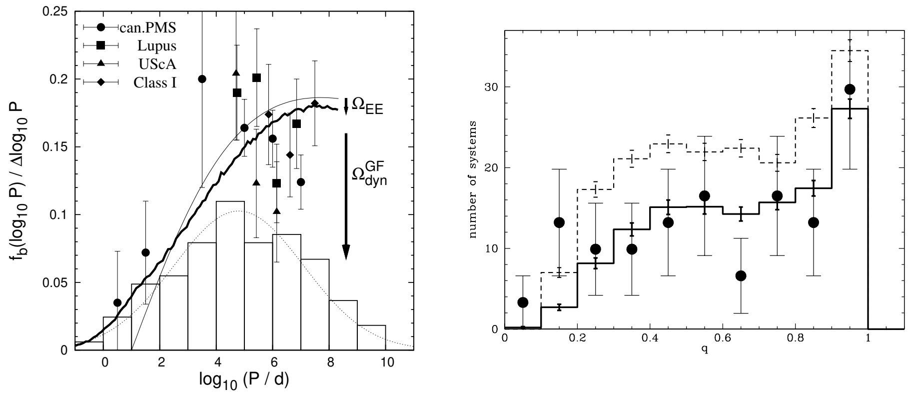

Figure 1. *Left panel*: The distribution functions of periods, $f_{\rm b} = f_{\rm bin}$, of about 1 Myr old late-type binary systems in star-forming regions as shown by solid symbols. The histogram shows the period distribution of main sequence G-dwarfs in the Galactic field (Duquennoy & Mayor 1991; Raghavan *et al.* 2010) which also matches that of K- and M-dwarfs that have a slightly lower binary fraction (see Fig. 2). For example, 8 per cent of all field-G dwarfs have a stellar companion with $10^3 < P/{\rm d} \leq 10^4$. The thin solid line is the birth distribution of periods, $f_{\rm P,birth}$ (Eq. 4). It evolves through pre-main sequence eigenevolution ($\Omega_{\rm EE}$) in about 10^5 yr per binary to the thick solid curve which is the initial period distribution function, $f_{\rm P,init}$. The stellar-dynamical encounters in the embedded clusters in which the stellar population is born in evolve (by the stellar dynamical operator, $\Omega_{\rm dyn}^{\rm GF}$) $f_{\rm P,init}$ to the observed period distribution function, $f_{\rm P,ms}$, in the Galactic field shown as the thin-dotted curve. $\Omega_{\rm dyn}^{\rm GF}$ is a function of the distribution of masses of the embedded clusters, $M_{\rm ecl}$, and their half-mass radii, $r_{\rm h}$. Adapted from fig. 1 in Marks, Kroupa & Oh (2011), for further details see there. *Right panel*: The mass-ratio distribution for primaries with masses $0.1 \leq m_1/M_\odot \leq 1.1$. The solid circles show the mass ratio distribution of the volume-limited Solar-neighbourhood sample of observed stars (Reid & Giziz, 1997). The dashed histogram is the initial ($f_{\rm q,init}$, Eq. 4) and the solid one is the model main-sequence Galactic-field population, $f_{\rm q,ms}$, computed according to Eq. 3. Note that the stellar dynamical processing in the birth embedded clusters reduces the number of binaries at all values of q and how the pre-main sequence eigenevolution leads to a near-perfect match between the observed peak and the model near $q \approx 1$ (the "pre-main sequence eigenevolution" peak in q). Adapted from fig. 17 in Kroupa (2008), for details see there.

already with fully code-integrated stellar evolution and the most advanced mathematical transformations of the equations of motions for efficient and accurate time integration). This development allowed the code to efficiently compute the dynamical interaction of compact sub-systems of evolving stars, and it just so happened to occur when the first high-resolution imaging and speckle interferometry observations of nearby star-forming regions done in the early 1990s (see

Kroupa 1995a for details) turned up the unexpected result that nearly all pre-main sequence stars there are binaries. I had witnessed these results when they were reported for the first time at a conference on binary systems in Atlanta in 1992, and it became apparent to me that the binaries probably break up in embedded clusters. But this needed to be shown with Nbody calculations. This is where the new chain regularisation came into play. The above demonstrates how different research developments can conspire to interfere constructively at just the right time to allow useful progress. The Aarseth suit of programmes (Aarseth 1999, 2003, 2008) contains a stand-alone fortran chain-regularisation code (chain.f) for integrating small-N systems, and given the importance of chain regularisation, a newer C-code, CATENA, was later developed by Jan Pflamm-Altenburg and applied to a Nbody study of the Trapezium star cluster (Pflamm-Altenburg & Kroupa 2006).

The procedure to derive the birth (and initial) distribution functions is as follows: the Galactic field population needs to be reproduced given the pre-main sequence constraints (e.g. Fig. 1). The constraint that $f_{\rm bin,birth} \approx 1$ for the pre-main sequence population stemming from the nearby (and thus common low-density) molecular cloud clumps suggests the birth period distribution function to be rising with P, since, by the Heggie-Hills law (Sec. 5) the close binaries are not significantly affected by stellar-dynamical encounters while the long-period ones are likely to be disrupted. Indeed, the data in the left panel of Fig. 1 tell us that the binary fraction for $P < 1000\,{\rm d}$ ($f_{\rm bin}(P < 10^3\,{\rm d}) \approx 13$ per cent) is similar in the pre-main sequence population as it is for the metal-rich Galactic-field population in the Solar neighbourhood. In order to arrive at a mathematical formulation of the plausible birth distribution functions, Kroupa (1995a) applied an iterative procedure. By performing Nbody simulations with an Aarseth (1999, 2003, 2008) code of star clusters until their dissolution (such that their stars become field stars) with different birth densities and starting with a first guess for $f_{\rm P,birth}, f_{\rm q,birth}, f_{\rm e,birth}$, the destruction of binary orbits through stellar-dynamical encounters was quantified. Sverre had to change the memory array structure in his Nbody software so I could do the computations of star clusters initially consisting of 100 per cent binary systems, a problem considered to be sacrilegious and heretical until then (Aarseth, 1993, private communication). For example, the seminal work on “The evolution of open clusters” by the PhD student of Sverre Aarseth, Elena Terlevich (1987), still relied on single-star simulations with an earlier Aarseth Nbody code. Returning to the present issue, a second round of simulations used improved $f_{\rm P,birth}, f_{\rm q,birth}, f_{\rm e,birth}$, with this being repeated until these distribution functions were consistent with the observational constraints evident in the left panel of Fig. 1 for the pre-main sequence stars and the field stars. “Consistent” here means according to inspection by eye. It would be wrong to attempt a formal fit, because the pre-main sequence observational data are for $\approx 1\,{\rm Myr}$ old systems that have already been dynamically processed to some degree despite stemming from low-density molecular clumps. And the Galactic field population stems from the superposition of many dissolved embedded clusters, the distribution of which is unknown. The choice of action was therefore to seek $f_{\rm x,birth}$ such that an overall good reproduction of the $\approx 1\,$Myr and $\approx 5\,$Gyr old populations could be achieved through Nbody modelling. An important cross check was to ensure that the computed mass ratio and orbital eccentricity distributions were also, at the same time, consistent with the data available for the pre-main sequence and the field binaries. *This uniquely demonstrated that the field population can be arrived at by the stellar-dynamical processing of the birth population of binaries that is consistent with the observed binary systems in a pre-main sequence stellar population. For this to work, stars must form as binary systems in embedded clusters, implying that embedded clusters are the fundamental building blocks of galaxies (Kroupa 2005).*

The next step was to account for the observed correlations between P, q and e for systems with $P \lesssim 10^3\,$d. These correlations are obtained from the birth distributions via *pre-main sequence eigenevolution* which models the dissipative evolution of these three internal degrees of freedom during the first $\approx 10^5\,$yr of a binary's existence. To achieve a useful mathematical description of this, theoretical work on the tidal dissipation and tidal circularisation was accommodated (Kroupa, 1995b). Pre-main sequence eigenevolution leads to the shrinkage of an orbit, to its circularisation and to a change in the mass ratio, q, as the secondary component in the binary accretes gas from the accretion disk of the primary. The changes in P, q and e occur in dependence of the birth P. The formulation of pre-main sequence eigenevolution was improved slightly by Belloni *et al.* (2017) who took into account the properties of stellar populations in globular star clusters. Pre-main sequence eigenevolution thus changes the masses of some of the stars and leads to mergers such that the *initial* population (after the $\approx 10^5\,$yr evolution) has a larger mass in stars by a few percent and contains a few per cent mergers of all initial stars which are now single stars.

The so-arrived at comprehensive and computationally highly efficient approach was tested against the field population of nearby (and thus fully resolved truly single) and more distant (and thus unresolved binaries) stars in terms of star counts, i.e. their respective stellar luminosity functions, by computing star clusters until their dissolution (Kroupa 1995c). An additional test of the above birth distribution functions plus pre-main sequence eigenevolution theory was performed against the observations of the density profile, the velocity dispersion profile, the IMF and observed binary-star properties in the Orion Nebula Cluster and the Pleiades cluster (Kroupa, Aarseth & Hurley 2001). These calculations also demonstrated how an embedded cluster emerges through gas expulsion from its molecular cloud clump to become part of an OB association and a long-lived open star cluster.

In order to allow the rapid synthesis of entire stellar-field populations in galaxies assuming all stars form as binaries and in embedded clusters, the above was formalised in terms of operators that act on $f_{\rm P,birth}, f_{\rm q,birth}, f_{\rm e,birth}$ (Kroupa 2002). The mathematical formulation of the operators was achieved by Marks, Kroupa & Oh (2011) in terms of the *pre-main sequence eigenevolution operator*, $\Omega_{\rm EE}$ and the *stellar-dynamical operator*, $\Omega_{\rm dyn}$. After applying $\Omega_{\rm EE}$ to evolve the birth to the initial distribution of binaries in one computational step, the population is stellar-dynamically processed by a second computational step by applying $\Omega_{\rm dyn} = \Omega_{\rm dyn}(M_{\rm ecl}, r_{\rm h})$ to obtain the dynamically processed initial population after a specified time. Because further dynamical processing of this remaining binary population largely halts after gas expulsion expands the cluster, freezing the stellar population and its binaries (apart from individual hardening events), the frozen population in a post-gas-expulsion young open cluster becomes its contribution to the Galactic field population.

In summary, we can write that at a given x, the operator $\Omega_{\rm dyn}$ reduces or increases $f_{\rm x}$ as a consequence of the stellar-dynamical processing in the cluster characterised by $M_{\rm ecl}, r_{\rm h}$, i.e. its density. We thus have the evolution of the birth population to the frozen main sequence (ms) one being computed via the following steps:

$$f_{\rm x,init}(x) = \Omega_{\rm EE}\, f_{\rm x,birth}(x) \implies f_{\rm x,ms}(x) = \Omega_{\rm dyn}(M_{\rm ecl}, r_{\rm h})\, f_{\rm x,init}(x)\ . \qquad (3)$$

Thereafter the same step as in the first part of Eq. 3 can be performed using the main-sequence version of $\Omega_{\rm EE}$ in order to correct the orbital elements of the short-period ms binaries for tidal circularisation (Kroupa, 1995b). The integration over (i.e. summation of) all such $\Omega_{\rm dyn}$-processed binary populations yields the field stellar population as it emanates from the population of embedded clusters that form at a given time. This is represented in Fig. 1 by $\Omega_{\rm dyn}^{\rm GF}$.

Therewith a formulation of the binary-star birth binary distribution functions that are valid for star formation in the Solar vicinity of the Galaxy is obtained,

$$\begin{array}{rl} \xi(m) & \text{eq. } 5-8 \text{ in Kroupa } et\ al. \text{ (2024)}, \\ f_{\rm P,birth}(q), f_{\rm q,birth}(P), f_{\rm e,birth}(e) & \text{eq. 3b} - 8 \text{ in Kroupa (1995b)}, \\ & \text{updated in Belloni } et\ al. \text{ (2017)}. \end{array} \qquad (4)$$

4. Using the initial distribution functions

The usage of the stellar IMF, $\xi(m)$, for various problems in galactic, extragalactic and high-redshift problems is well documented (Hopkins 2018; Kroupa *et al.*, 2024).

Similarly, with the birth distribution functions, $f_{\rm P,birth}(q)$, $f_{\rm q,birth}(P)$, $f_{\rm e,birth}(e)$, in hand and assuming these to be invariant to the physical conditions, (Carney *et al.* 2005; Kroupa 2011) any population of any age can be synthesised. For example, *forbidden binaries* must exist (Kroupa, 1995b) – these are binaries that have been hardened through an encounter and appear above the eccentricity–period cutoff that arises from tidal circularisation. Such forbidden binaries circularise on the orbital time scale and allow insights into stellar interiors.

We can calculate the evolution of the birth binary population in the Orion Nebula Cluster and the Pleiades (Kroupa, Aarseth & Hurley, 2001). The initial conditions under which the globular cluster ω Cen, with its very small present-day binary fraction (Wragg *et al.*, 2024) and its associated retrograde field population (Carney *et al.*, 2005) most likely formed in can be inferred (Marks, Kroupa & Dabringhausen, 2022). This system is interesting, because Carney *et al.* (2005) note that the prograde metal-poor population of stars has an observed binary fraction that is not distinguishable from that of the metal-rich population, while the retrograde field population co-orbiting with ω Cen and of the same low metallicity has a much smaller binary fraction. These differences can be understood if the retrograde population was born in one or more dense star-burst clusters, the remnants of which today are evident as the ancient ω Cen. The implied high birth density of ω Cen of $\approx 10^8\, M_\odot/\mathrm{pc}^3$ (see fig. 10 in Marks *et al.* 2011) is of relevance also for the massive globular cluster 47 Tuc which is reported to likewise have an extremely low present-day binary fraction (Müller-Horn *et al.*, 2024).

Further, the relevance of star-cluster-induced binary-star mergers for the population of B[e] stars can be assessed: about 26–30 per cent of all O-type stars, 13-24 per cent of all B-type stars and 5-8 per cent of all A-type stars would be mergers (Dvořáková *et al.*, 2024). A substantial fraction of massive stars are thus probably mergers that were induced to happen through stellar-dynamical encounters in their young star clusters where they were born. The birth binary fraction of unity leads to profuse stellar-dynamical ejections of O and B type stars from their birth embedded clusters as shown by Nbody calculations (Oh & Kroupa, 2016) and as is now observed to be the case based on Gaia data (e.g. Stoop *et al.*, 2024; Carretero-Castrillo *et al.*, 2024).

We can also synthesise galactic field populations by adding the many contributions from all embedded clusters that have since dissolved leading to the prediction that star-forming late-type dwarf galaxies will have an overall binary fraction amongst its late-type stars of $f_{\mathrm{bin,ms}} \approx 0.8$, Milky-Way-type galaxies have $f_{\mathrm{bin,ms}} \approx 0.5$ (as is observed) and massive elliptical galaxies have $f_{\mathrm{bin,ms}} \approx 0.35$ (Marks & Kroupa, 2011). The dynamical evolution of the Cepheid population in a galaxy can be computed, giving insights, for example, into out-of-age Cepheids which are like blue stragglers and form after a binary system merges into the Cepheid instability strip (Dinnbier, Anderson & Kroupa, 2024).

Most of the above results relied heavily on the Aarseth Nbody codes to do the time-forward integrations of the equations of motions of the systems being studied. But these are CPU-time consuming and the development of the Ω-operators based on the Nbody results lead to the BIPOS1 programme. The programme BInary POpulation Synthesis 1 (BIPOS1) has been made publicly available to allow the rapid computation of dynamically processed stellar populations (Dabringhausen, Marks & Kroupa, 2022). The modelled population of galactic field stars turns out to be in excellent agreement with the observed population of M-, K- and G dwarfs (see Marks & Kroupa, 2011; Cifuentes *et* *al.* 2024), accounts for the peak at $q \approx 1$ (Fig. 1) *after* the above theory was formulated, and also reproduces the correlation between the binary fraction and the mass of the primary star observed for Galactic field stars (Fig. 2).

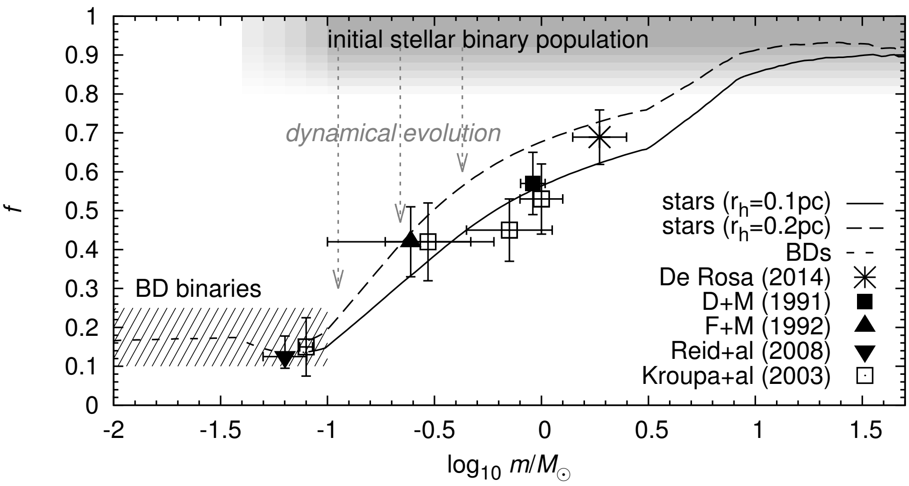

Figure 2. The binary fraction as a function of the primary mass. Stars are born in embedded clusters with half mass radii $r_{\rm h}$ as binaries. This birth stellar binary population has $f = f_{\rm bin,birth}(m_1) \approx 1$ *for all primary masses*, m_1. The stellar-dynamical encounters within the embedded clusters decrease $f_{\rm bin,birth}(m_1)$ more for smaller m_1 because less-massive binary systems are, on average, less bound. The calculations (via $\Omega_{\rm dyn}$) show the decrease to match the observed data of old Galactic field populations (depicted by the different symbols) very well. Brown dwarfs (BD) follow fundamentally different distribution functions to those of stars, and massive stars ($m_1 > 5\,M_\odot$) likewise have different pairing rules but appear to be born typically in triple and quadruple systems. This figure demonstrates that it is incorrect to take the observed decrease of the binary fraction with decreasing primary-star mass as a direct constraint on star-formation theory. Adapted from fig. 6 in Thies *et al.* (2015), for details see there.

5. The physical variation of the birth/initial distribution functions

The variation of $\xi(m)$ with metallicity, Z, and density of the embedded cluster is reasonably well understood (Kroupa *et al.*, 2024). From the empirical evidence it transpires that the gravitationally unstable gas clump which forms an embedded cluster does what theory broadly expected all along: in broad and simple terms, at high (e.g. Solar) metallicity, the collapse cannot be as deep because slightly overdense gas cools rapidly and so fragments into typically less massive proto-stellar cores, while at low metallicity, the collapse is deeper reaching larger densities before individual proto-stars emerge thus leading to a more top-heavy stellar IMF in the embedded cluster that forms in the molecular cloud clump.

The three birth binary star distribution functions, $f_{\rm P,birth}, f_{\rm q,birth}, f_{\rm e,birth}$, most likely vary in an according and a coordinated fashion. In extracting their variation from the available data, one needs to take into account that a metal-poor gas cloud will collapse to a denser state than a metal-rich one such that the ensuing low-metallicity embedded cluster will lead to a lower overall binary fraction *if* the birth distribution functions were to be invariant, because more of the binary systems are disrupted in a denser embedded cluster. Since *hard* (i.e. close, see below) binaries have too small a cross section for significant stellar-dynamical processing, the number of these does not change significantly and this is independent of metallicity. *Soft* (i.e. wide, see below) low-Z binaries would however be depleted compared to their counterparts at $Z_\odot$. According to Eq. 1, the binary fraction for close binaries would thus be slightly smaller at low than at high metallicity but would be quite insensitive to Z, while the binary fraction of wide binaries would be smaller at low Z than at $Z_\odot$. The data by Lodieu *et al.* (2025) may be consistent with this interpretation – see below.

However, more likely $f_{\rm x,birth}(\text{low}\, Z)$ will differ from $f_{\rm x,birth}(\text{high}\, Z)$ by the same broad physics: the proto-stellar core collapses to a higher density before two proto-stars emerge probably leading to a higher fraction of close binaries at low metallicity, as discussed by Mazzola *et al.* (2020). Longer-period binaries may appear just as in the high-Z case through random associations in the molecular cloud filaments of protostars for late-type stars (see Kroupa *et al.* 2024 for a discussion). Since the molecular cloud core evolves to a denser state at low Z, it is probable that also the wider binaries will end up having shorter P values than their equivalent high-Z counterparts.

The hard/soft boundary is of importance here: *hard binaries* have orbital velocities that are larger than the bulk velocity dispersion in the embedded cluster in which they are born, while *soft binaries* have slower orbital velocities. *There is no universal hard/soft boundary apart from that for an embedded cluster at a given time.* The hard/soft boundary is proportional to $r_{\rm h}^{-1/2}$ (Eq. 2) such that this boundary is at a higher velocity for a low-Z embedded cluster than for a high-Z cluster of equal mass. The binary-star semimajor axes, a, are also probably systematically smaller at low-Z than at high-Z, scaling proportionally to $r_{\rm h}$. But because the orbital velocity of a binary system is proportional to $a^{-1/2}$ (eq. 142 in Kroupa 2008), the fraction of hard to soft binaries may remain the same in an embedded cluster of low metallicity compared to an equal-mass cluster at high metallicity. However, it may be, by the Heggie-Hills law ("*hard binaries harden, soft binaries soften*", Heggie, 1975; Hills, 1975), that the denser environment in the metal-poor embedded cluster hardens the tight binaries such that the fraction of *close* binaries ends up being larger in the metal-poor population than in the metal-rich one.

The above discussion may be relevant for the discovery through large surveys by Moe, Kratter & Badenes (2019) and Mazzola *et al.* (2020) that iron-poor populations have a higher fraction of *close* binaries than metal-rich populations. These authors also find the fraction of close binaries to vary with the alpha-element abundances. The variation of the observed populations with the metal abundance thus appears to be complicated. However, what matters is that the cooling function of the gas depends primarily on the available atomic transitions such that Z is the relevant quantity. Thus, a population that is alpha-element enriched but iron poor might have the same binary star properties to a population that is iron rich and alpha-element poor. For example, according to Forbes *et al.* (2011, see also Wirth *et al.*, 2022), $[Z/H] = [Fe/H] + [\alpha/H]$. Returning to the samples by Moe *et al.* (2019, their fig. 3) and Mazzola *et al.* (2020, their fig. 5), their data appear to indicate that at near Solar metallicity ($[\mathrm{Fe/H}] \approx 0$) the fraction of close ($P < 10^4\,\mathrm{d}$) binaries in a survey amounts to about 20 per cent which is consistent with Fig. 1 (left panel). Fig. 3 in Moe *et al.* (2019) suggests that at $[\mathrm{Fe/H}] \approx -3$ this fraction increases to 60 per cent. In Fig. 6 in Mazzola *et al.* (2020) the published fraction reaches > 100 per cent at moderate low metallicity ($[\mathrm{Z/H}] \approx -0.8$), leaving no room for a contribution to the stellar population by wider binaries (with $P > 10^4\,\mathrm{d}$). In contrast to the above studies, Lodieu *et al.* (2025) find Solar-type stars at low "metallicity" ($[Fe/H] < -1.5$) to have a comparable/much smaller binary fraction to those at Solar metallicity for close binaries ($< 8\,\mathrm{AU}$)/wide binaries ($8 - 10^4\,\mathrm{AU}$), respectively. In view of the above results that appear to differ substantially in terms of the reported binary fraction of close binaries at low Z, it is therefore unclear how these quoted fractions are to be compared and care needs to be taken in precisely defining what is counted how.

The above information provides a suggestion how the current formulation of $f_{\mathrm{x,\,birth/init}}(Z \approx Z_\odot)$ given by Eq. 4 can be generalised to other values of Z. An iterative process of matching the observational constraints (e.g. by Carney *et al.*, 2005; Moe *et al.*, 2019; Mazzola *et al.*, 2020; Lodieu *et al.*, 2025) to Nbody-generated data, similarly to that performed by Kroupa (1995a,b), will be necessary to infer a viable model. Hereby smaller r_{h} values at low Z will need to be probed. The so-arrived at model for $f_{\mathrm{x,\,birth/init}}(Z)$ will then be of importance to test and check the theory of star formation (rather than the other way around).

In performing such tests of star-formation theory, it needs to be remembered that computer simulations of collapsing molecular cloud cores do not produce a measurable $f_{\mathrm{x,\,birth/init}}(Z)$ for the same reasons why these functions cannot be measured on the sky (the forming binaries break up due to encounters in these simulations while others are still forming). The theoretically produced $f_{\mathrm{x,\,birth/init}}(Z)$ can however be extracted by tracing the individual stellar particles back to where and in which stellar system they were born in, in order to quantify as well as is possible the values of P, q, e for this system. The theoretical $f_{\mathrm{x,\,birth/init}}(Z)$ can be constructed from the distributions of the so obtained quantities.

Metallicity and density dependent operators $\Omega_{\mathrm{EE}}(Z), \Omega_{\mathrm{dyn}}(r_{\mathrm{h}}, M_{\mathrm{ecl}} : Z)$ will then need to be quantified along the lines as performed by Marks, Kroupa & Oh (2011) to allow rapid dynamical synthesising of stellar populations being born in galaxies of different properties and at different redshifts as probed by Marks & Kroupa (2011) for the $Z \approx Z_{\odot}$ case.

6. Conclusions

The difference between the binary populations in star forming regions and the Galactic field can be understood if stars form mostly as binary systems and in embedded star clusters in molecular cloud clumps. Dynamical processing in these embedded clusters dissolves a large fraction of the soft binaries. The binary properties of Galactic field stellar populations are therewith reasonably well understood at Solar metallicity.

The Sun too may have had a binary companion orbiting it with a birth period $P \gtrsim 10^6\,\mathrm{d}$ ($a \gtrsim 300\,\mathrm{AU}$) in order to be consistent with the current orbit of Neptune. Such orbits in fact dominate the birth period distribution function (Fig. 1). The existence of planetary systems around most stars is thus not in contradiction to all stars forming as binary systems. The finding that many exoplanetary systems appear to be "jumbled-up", i.e. not as regular and orderly as our Solar System (e.g. Zhu & Dong, 2021), may be due to the perturbations during the formation stage or thereafter by the not so distant companion star and stellar-dynamical encounters in their birth embedded clusters (Thies *et al.* 2011).

There is evidence that low-metallicity populations follow the trend already well documented by the variation of the stellar IMF. The sense may be that with decreasing metallicity the average stars appear to be more massive and consist of closer binaries at birth, and probably also with more similar companion masses. There is thus emerging evidence how the three birth distribution functions, $f_{\mathrm{P,birth}}(P : Z, m_1), f_{\mathrm{q,birth}}(q : Z, m_1), f_{\mathrm{e,birth}}(e : Z, m_1)$, differ form their metal-rich ($Z \approx Z_{\odot}$) counterparts such that, upon the dynamical processing in the low-metallicity embedded clusters, the survey results are obtained. However, as discussed above, some of the evidence appears to be contradictory.

In terms of challenges for the future to improve the mathematical description of birth distribution functions of stars: (A) the variation of these with metallicity and density of the embedded clusters in which stars are born will need to be studied. This will require Nbody simulations to quantify the stellar-dynamical processing of the distribution functions for plausible mass functions of embedded clusters that are likely to have spawned the stellar populations under study. For example, a field population might stem from one single massive star burst cluster or from a large population of less-massive embedded clusters. An example of this is the work on ω Cen and its associated field population mentioned in the text. (B) The extension of the birth distribution functions to include triple and quadruple systems is needed. This is an interesting task to be mastered by seeking a mathematical description that preserves the above (Eq. 4) but accommodates additional companions without violating the stellar IMF.

As a final comment, the above and associated work shows that there is no single correction for unresolved binary systems for a given star-count survey because the dynamical histories of observed populations can differ such that their binary properties differ. For example, a low-mass molecular cloud in which low-mass embedded clusters emerge spawns a T Tauri association with a high binary fraction ($f_{\rm bin} \approx 0.8$). A neighbouring massive molecular cloud in which ONC- (or even NGC3603-) type star-burst embedded clusters condense spawns an OB association with an overall binary fraction $f_{\rm bin} \approx 0.4 - 0.5$. And, even if they are merely "hilfskonstrukts", the inference of the true birth/initial distribution functions are needed for efficiently modelling stellar populations. Finally, given some observational survey data, understanding the underlying stellar populations and their binary properties non-negotiably requires full understanding of their stellar-dynamical and astrophysical evolution. Without the monumentally fundamental Nbody-development work by Sverre Aarseth in Cambridge since the 1960s this would not have been possible to be achieved.

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