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Doubly eclipsing systems: Divide et impera

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Abstract. Multiply eclipsing systems are still quite new type of objects, but their number is still increasing: now reaching almost 1 thousand. They represent rather challenging task for modelling. Even in the case when having enough data for a complex analysis. There are many effects acting together, which have to be taken into account. However, for most of them still the dynamical interactions are small enough. Hence, at first, the systems can be considered as two independent triple systems. And for the analysis to use a well-known old method : Divide et impera.

Key words: binary stars – multiple systems – photometry

1. Introduction

The classical role of eclipsing binaries as a standstone of modern astrophysics was discussed elsewhere (e.g. Southworth 2012). However, nowadays the amount of detected eclipsing binaries reached a number of more than 2 millions. Hence, it is absolutely beyond the capacity of human beings to be able to analyse each particular binary in detail. Only some statistical properties of them are being studied sometimes.

Another way of thinking about the topic would be to focus on some very specific type of eclipsing systems, which still deal with much lower numbers. Objects observable, but rare enough, which are of high astrophysical importance. The multiply-eclipsing systems are definitely such type of objects. With these I mean those stars, where one can see different periods of various eclipses, where one periodicity obviously cannot easily explain all of the observed eclipses. Therefore, such systems can be either triply eclipsing triples, where the third body moves on an outer orbit (also typically co-planar) to the inner eclipsing pair causing the tertiary eclipses (of typically quite complicated shape). Or another scenario where we have a bound 2+2 system of two eclipsing inner binaries, which do not have to necessarily eclipse each other on their mutual orbit. Hence, the two eclipsing periods typically detected via photometry make them a so-called "doubly eclipsing system", i.e. two eclipsing periods apparently coming from one point source on the sky. We should more correctly classify them only as candidates, because their mutual orbit is not known in general. However, the non-detection of any mutual movement of the two pairs around each other does not prevent us to classify these objects as doubly eclipsing.

The vast majority of such systems were discovered quite recently only due to continuous monitoring of the sky with different photometric surveys like OGLE (for example 32 detected doubly eclipsing binaries from the OGLE fields towards the Galactic Bulge, see Soszyński et al. 2016), or satellite telescopes like Corot, Kepler or recently TESS (about three hundred new systems detected by Kostov et al. 2022, Zasche et al. 2022b, and Kostov et al. 2024). Quite surprisingly, 20 years ago nobody in the astrophysical community knew anything about such kind of objects, the first one was V994 Her discovered by Lee et al. (2008). Nowadays, the number of candidates with two distinctive photometric periods reaches one thousand (980 published yet, state for the date of October 2024). Among these systems I count only these ones, which have both eclipsing periods published and are obviously not blends coming from two distinct close-by sources on the sky.

2. Methods for analysis

In principle, there are two possible methods for analysing these complicated systems.

1. Complete combined dynamical modelling of the whole 2+2 system. Joining all individual data into one dataset and running the full dynamical computation and modelling of LC, RV, ETV, or SED together. Process time-consuming and non-trivial, and demanding good starting values of parameters and well-observed data. This approach was being used by a group of people around T.Borkovits, and their analysis led to many interesting results on several dynamically interacting systems, e.g. Borkovits et al. (2021), or Pribulla et al. (2023).

2. Another possible approach would be used only for weakly bound quadruples, where their outer orbital periods are longer compared to the inner periods. It means where the mutual dynamical interaction is smaller and one can use some simplified approach via linearization of the whole modelling. I call this approach as "divide et impera". It means cutting the whole problem into smaller pieces. And then to solve the individual datasets independently with some simplified assumptions. In the following sections, I will present why such an approach is mostly sufficient.

Linearization in the problem would be to use linear ephemerides instead of complicated approach of ETV variation described with LITE or dynamical effects. Such an ETV is not known in the beginning and one has to use some preliminary assumption of the correct period for example. But with a linear ephemerides one can fit the individual light curves and radial velocity curves and then season by season to join them together to some joint approach. For systems where the long mutual period A-B is long enough, such an approach is justifiable, as can be seen in our illustrative figure 1.

One can use such a method only when the outer period is long enough, and the dynamical interactions are generally small (the ratio of P_{AB}/P_A or P_{AB}/P_B large enough). Another necessary condition is that the number of datapoints in individual datasets is large enough when cutting the whole dataset into shorter intervals.

Here comes quite important point of our whole approach. Judging whether our method is adequate, or the ratio of periods is small enough should be done star-by-star by the one doing the analysis. For sure, the outer period should be an order of magnitude longer than both inner periods. And also the number of data points in every single cutted interval should be enough (at least dozens of photometric data points for each light curve).

Another step in the method also using this "divide et impera" approach would be for the disentangling of the combined light curve into the two individual light curves of both A and B pairs. Using a similar assumption, we shall use some preliminary fit of LC of pair A, subtract it from the combined photometry, and do the same with LC of pair B. Doing this procedure several times iteratively, one gets the final disentangled light curves of both A and B pairs. However, also here the simplification of the process is obvious - one has to have a linear ephemerides, and both light curves should be constant in shape over the time interval. When such an assumption is not fulfilled, then again one has to cut the whole interval into smaller pieces, for which the assumption is acceptable.

3. Usage of the method

One can ask whether the presented method is adequate for most of the doubly eclipsing 2+2 quadruples. Or it is too simplifying and can be used only for several systems. I will show that it is usable for most of the systems known nowadays.

What is the crucial parameter for judging whether the linearization is acceptable or not is the period of the outer orbit, or the ratio of inner to outer period. Because most of the studied systems detected as doubly eclipsing have inner orbital periods of days typically, the outer period of years or decades is certainly long enough. It means that the dynamical interactions would be small enough, and for example only the geometrical LITE can also be used for fitting of the ETV curve. On the other hand, when we deal with an outer period of hundreds of days like for BU CMi, where the outer period is only of 121 days (Pribulla et al., 2023), this simplification definitely cannot be used.

Therefore, for the decision on what is the fraction of all doubly eclipsing stars which can be studied in our way, I collected the published systems. This means that only these 2+2 quadruples definitely proved to orbit, where their mutual

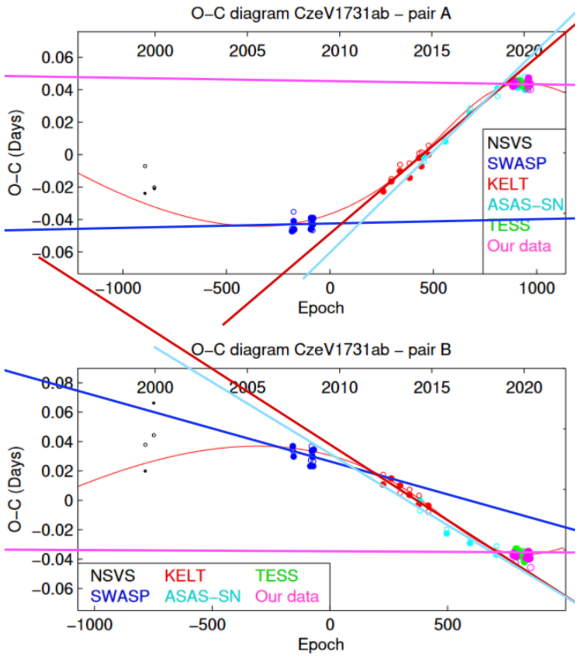

Figure 1. The ETV diagram of both A and B pairs of system CzeV1731. All individual datasets were analysed in some simplified approach with linear ephemerides, which were different for different time epochs.

orbit was derived. Surprisingly, among the known 980 doubly eclipsing systems still only 58 of them were published as bound 2+2 with derived orbits. These mutual-orbit solutions were published in these papers: Zasche & Henzl (2022), Rappaport et al. (2017), Zasche et al. (2022a), Zasche et al. (2024a), Powell et al. (2021), Kostov et al. (2021), Zasche et al. (2019), Pejcha et al. (2022), Rowden et al. (2020), Pribulla et al. (2023), Zasche et al. (2023b), Kostov et al. (2024), Rappaport et al. (2016), Kostov et al. (2023), Zervas et al. (2024), Ádám et al. (2023), Hajdu et al. (2019), Kővári et al. (2024), Zasche et al. (2023a), Borkovits et al. (2018), Zasche et al. (2020), Borkovits et al. (2021), and Zasche et al. (2024b).

As is clearly seen in our figure 2, most of the studied systems by our group are these ones having the outer periods of years or decades. It means those, where their dynamical interaction is in general very small. On the other hand, the group around T.Borkovits using their sophisticated dynamical modelling code sometimes deal with very dynamically interacting systems with much shorter outer periods. From the total 58 systems with derived orbits, only 4 have $P_{AB}/P_A < 100$, and only 6 have $P_{AB}/P_B < 100$.

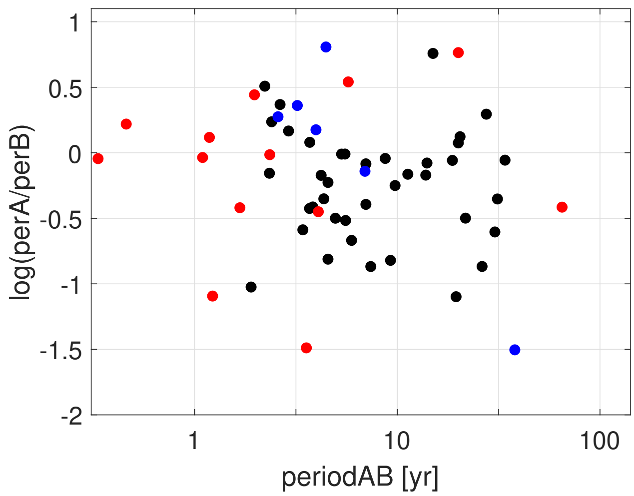

Figure 2. Plot of the doubly eclipsing systems with published outer mutual orbit of both pairs (pair A and pair B around a common barycenter). Distinction according to the author of the study for a particular system: black dots our systems, red dots by T.Borkovits et al., and blue dots by the other authors.

Worth of mentioning is a fact that apart from the number of data points and ratio of periods, there are also some other limitations for using our simplified approach. Cutting the whole interval into smaller pieces is one way, but the constancy of the shape of the light curve in every subset is an important task. The shape of the light curve cannot vary in time (like typical spot modulation for some contact binaries or late-type stars), or we should use multiple light curve templates over the whole time interval. The same applies also for the case where the inclination of either of the binaries is not constant in time. A similar problem should arise when rapid apsidal motion appears, and this effect changes the shape of the light curves when comparing the season-to-season data. Quite challenging would also be these systems, where some rapid ETV changes appear due to very eccentric orbits and the quadruple-star interactions, which can occur also on orbits with longer mutual periods, see e.g. Borkovits et al. (2015).

4. Conclusion

To summarize, I have shown that the simplified method of cutting the whole dataset into smaller pieces and using some assumptions of linear ephemerides is appropriately usable for the majority of known bound doubly eclipsing systems yet. On the other hand, probably the most interesting dynamically interacting quadruples on shorter orbits should be studied with a complete photo-dynamical approach. And especially these systems (sometimes called as celestial mechanics laboratory) can give us some clue about the true origin of these complicated 2+2 quadruple systems.

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