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New "pseudo-rotating" model atmospheres

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Abstract. Regarding model atmospheres, rotation effects are mostly neglected, and, at best, the rotational broadening correction is being applied to a non-rotating synthetic spectrum. However, this approximation does not work for fast rotators ($v_{\rm eq} > 200\,{\rm km\,s^{-1}}$), where the effect of gravity darkening starts to be prominent. The need for this correction starts to be significant for stars hotter than F7. We introduce a new model atmosphere grid covering the region of the main-sequence A- and B-type stars. The model allows more robust follow-up studies, mainly to provide an alternative method to determine the inclination angle of the rotational axis by inversion. Calculating synthetic magnitudes and colors helps constrain the Main Sequence's rotational broadening and determine other biases related to the stellar rotation that influence the parameters determined from CMD. Determining rotational parameters from gravity darkening provides an alternative way of disentangling $v \sin i$ values and leads to a more precise understanding of the distinct populations of rotating stars.

Key words: stars: rotation – stars: atmospheres – stars: fundamental parameters

1. Introduction

The gravity-darkening law, in general form, is given by equation (1), where the exponent β can equal either 1 (von Zeipel law), 0.32 (convective envelope), or other value for the mixed case.

$$F = \sigma T_{\rm eff} \sim C(\tilde{\omega}) g^{\beta} \tag{1}$$

Values of β exponent values were determined by Claret (1998, 2000, 2003). Integrating the surface gravity over Roche surface leads to the equation (2) for the co-latitudinal gradient of effective temperature.

$$T_{\rm eff}(\theta) = \frac{L}{4\pi\sigma R_{\rm pole}^2} t_n^4(\theta) \tag{2}$$

With given $T_{\rm eff}(\theta)$, $g(\theta)$ and a vector towards the line of sight now, we can solve radiative transfer for each (θ, φ) point on the surface to get specific intensity $I_\nu(\mu)$. Consequently, we can integrate intensity over the surface to get total luminosity radiated in a line of sight (3).

$$\mathcal{L}_\nu(\tilde{\omega}, i) = \int_A I_\nu(\mu)\,\mu\,dA \tag{3}$$

The $\mathcal{L}_\nu(\tilde{\omega}, i)$ is a synthetic spectrum of a rotating star, not yet corrected for rotational broadening.

To achieve more robust results, Espinosa Lara & Rieutord (2011) assumed a generic function (4) that gives the flux of the rotating star.

$$\mathbf{F} = -f(r,\theta)\mathbf{g}_{\text{eff}}, \tag{4}$$

It is safe to assume that the effective surface gravity g_{eff} is very close to the Roche surface and thus defined by (5).

$$\mathbf{g}_{\text{eff}} = \left(-\frac{GM}{r^2} + \Omega^2 r \sin^2\theta\right)\mathbf{e}_r + (\Omega^2 r \sin\theta\cos\theta)\mathbf{e}_\theta \tag{5}$$

With help of the substitution (6) it is possible to derive (7).

$$\cos\vartheta + \ln\tan\frac{\vartheta}{2} = \frac{1}{3}\left(\frac{\Omega}{\Omega_{\text{k}}}\right)^2\left(\frac{r}{R_{\text{e}}}\right)^3\cos^3\theta + \cos\theta + \ln\tan\frac{\theta}{2} \tag{6}$$

$$T_{\text{eff}} = \left(\frac{F}{\sigma}\right)^{1/4} = \left(\frac{L}{4\pi\sigma GM}\right)^{1/4}\sqrt{\frac{\tan\vartheta}{\tan\theta}}\,g_{\text{eff}}^{1/4} \tag{7}$$

The equation (7) represents a more robust gravity darkening law which is not dependent on the free β parameter.

2. Motivation

Compared to non-rotating stars, the spectrum is altered not only in the continuum but also in the shape of the spectral lines as shown already by Maeder & Peytremann (1970). The luminosity of a fast rotator is slightly lower than that of the non-rotating counterpart; however, the mean effective temperature appears to be significantly lower. Fast rotators appear redshifted on an H-R diagram relative to slow rotators with the same composition and mass. At the same time, the apparent effective temperature strongly varies with the inclination of the rotation axis. A rotating star, oriented pole on, appears hotter than the same star oriented perpendicular to the line of sight.

Thanks to CHARA/MIRC, we can perform direct imaging of the rapid rotators (see Fig. 1). On the contrary to what was anticipated, the gravity-darkening exponent β appears to be significantly lower than 1 for A- and B-type stars (Monnier et al., 2014). This suggests that performing follow-up research via indirect methods on a larger sample is vital.

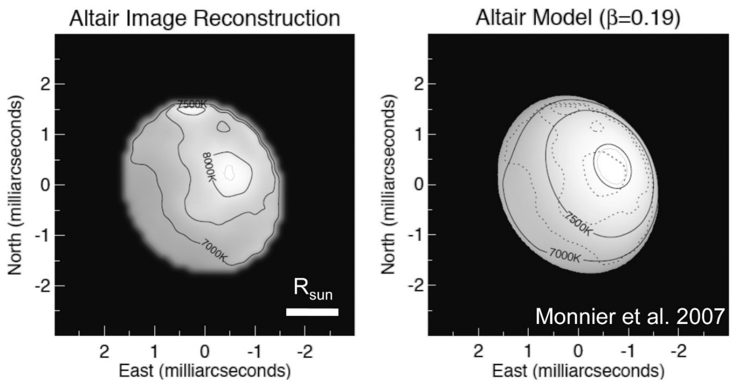

Figure 1. First image of a main-sequence star other than Sun compared to a model (Monnier et al., 2014).

The inclination of the rotational axis of individual stars is mostly unknown. The construction of a model atmosphere grid covering A- and B-type MS stars provides the possibility to perform an inversion to determine the previously unknown inclination of the rotation axis.

Since the position of rotating stars on an H-R diagram depends on rotational velocity and axial tilt, neglecting the population of the fast rotators in a star cluster could undermine the cluster age and distance determination given by isochrone fitting.

3. Method

The equation (7) allows us to calculate the surface grid of the $T_{\rm eff}$ and $\log g$ values. To calculate the specific intensity in the line of sight of the individual surface points, we use ATLAS12 code (Kurucz, 2013) for A-type stars and cooler B-type stars, and TLUSTY code (Hubeny et al., 2021) for hotter B-type stars. Since the resulting spectra result from specific intensity integration, there is no need to do limb darkening correction. Nevertheless, the resulting spectra would still require rotational broadening correction. As explained by Pérez Hernández et al. (1999), rotational broadening does not change equivalent width, so passband convolution can be safely calculated *before* applying the rotational broadening correction; however, to get good high-resolution synthetic spectra, the rotational broadening has to be also calculated implicitly, as the result of the surface synthesis (Montesinos, 2024). The recent versions of our code apply rotational Doppler correction to each individual surface element.

The synthetic spectra are then used to calculate synthetic magnitudes in selected photometric systems (Johnson-Kron-Cousins, Strömgren, Geneva, SDSS,

Table 1. Parameter space of the calculated model atmosphere grid.

Parameter	Range	Step
effective temperature	$10\,000\,\mathrm{K} \leq T_{\mathrm{eff}} \leq 30\,000\,\mathrm{K}$	$200\,\mathrm{K}$
surface gravity	$1 \leq \log g \leq 5$	$1\,\mathrm{dex}$
metallicity	$+0.5 \leq [\mathrm{M/H}] \leq -2$	$0.5\,\mathrm{dex}$
microturbulence	$0\,\mathrm{km\,s^{-1}} \leq \xi \leq 4\,\mathrm{km\,s^{-1}}$	$1\,\mathrm{km\,s^{-1}}$
equatorial velocity	$150\,\mathrm{km\,s^{-1}} \leq v_{\mathrm{eq}} \leq 250\,\mathrm{km\,s^{-1}}$	$50\,\mathrm{km\,s^{-1}}$
inclination	$5^\circ \leq i \leq 90^\circ$	$5^\circ, 10^\circ$

Gaia) to investigate changes in photometric characteristics caused by rotation (as demonstrated by, e.g., Lazzarotto et al., 2023).

4. Challenges

The parameter space of the new model atmosphere grid is constrained by the boundaries defined in Table 1.

This means we have 100 points in effective temperature, 20 points in surface gravity, 5 points in metallicity, 5 points in microturbulence, 6 points in equatorial velocity, and 10 points in inclination. In addition to that, each combination of parameters integrates over approx. 10^4 surface points. As a result, we need to perform 10^{10} model atmosphere calculations to complete the grid. In order to get at least close to this goal we needed to parallelize all computational tasks that are independent. At the same time, we managed to build up and maintain a dedicated computing cluster to fulfill this goal.

5. Conclusion

Construction of model atmospheres for A- and B-type stars incorporating gravity darkening with correction corrected by exponent (Claret, 1998, 2000, 2003) can lead to slightly exaggerated gravity darkening gradient. Comparison with the observing data suggests that the approach used by Espinosa Lara & Rieutord (2011) leads to more realistic results. We are actively working to reach full parameter space coverage, which will open up new possibilities for follow-up studies. Especially in the case of fast rotators, it can give us the more precise position of a star on an H-R diagram, leading to more accurate stellar parameters. The possibility of inverse calculation to determine the axial tilt of individual stars enables us to perform an extensive study of the inclination distribution in the Milky Way Galaxy and the Magellanic Clouds.

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